UNDERGRADUATE STUDENTS' MENTAL MODELS OF ELECTROSTATIC POTENTIAL

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Electromagnetism (EM) is considered as one of the essential components of core physics course where 'Electrostatics' lies at its foundation. 'Electrostatic Potential' is one of the most crucial concepts introduced in electrostatics connecting most fundamental physical quantities namely, electrostatic field and potential energy. Therefore, clarity in understanding of electrostatic potential is crucial for interaction of charges (or charge distributions) with an electric field. Since it is presented at undergraduate level in a highly mathematical and abstract framework, there remains scope for development of students' mental model which are divergent from the well-negotiated scientific concepts. Therefore, in this study, we have made an attempt to explore students' mental models for the concept of 'Electrostatic Potential by using problem solving tasks and interviews. We have found significant alternative conceptions which pose a challenge for meaningful learning.

INTRODUCTION

Electromagnetism (EM) is amongst the core subjects in undergraduate physics curriculum and plays a pivotal role in the development of many applied as well as interdisciplinary areas in physics. At the undergraduate (UG) level, EM is introduced conceptually with electrostatics in a highly abstract and mathematical framework (Griffiths, 2013). Electrostatics is substantially less intuitive than classical mechanics (which is introduced first at the UG level) and therefore, in addition to the common sources of misconceptions borrowed from mechanics, there is a variety of sources which are related to the abstract new concepts of electrostatics (as well as magnetostatics). In fact, conceptual development of electrostatics through experiments at an undergraduate level is a rare occurrence (Dvořák, 2012). There are a very few 'classroom environment' experiments in electrostatics through which, quantifiable information could be derived (Dvořák, 2012; Liengme, 2014). Hence, the basis of certain assumptions as well as 'laws' in electrostatics could be interpreted by students in varied manner when the concepts are introduced to them. In addition, the complex nature of mathematically defined quantities (including vectors, complex numbers, derivatives etc.) in electrostatics results in unforeseen cross-linkages in conceptual framework and consequently, students may have varied mental models at the undergraduate level. Within the constructivist framework those mental models which are divergent from well negotiated scientific concepts of the given time can be referred to as alternative conceptions (AC). Such AC in the area of electrostatic force/field and current electricity have been investigated by a few groups across the globe, essentially from an application perspective (Bull, Jackson & Lancaster 2012; Aubrecht & Raduta, 2005; Baybars, 2019; Samsudin et al., 2019). In one of the recent studies, Lindsay

(2014) presented an exploratory investigation on UG students' conceptions about electrostatic and gravitational potential energy which is crucial for making sense of energy (potential) in chemistry as well as biology. In electrostatics, potential energy in charged systems is essentially derived from basic understanding of electrostatic potential and field. In fact, electrostatic potential (scalar) is one of the most-used physical quantities in electrostatics as it connects all the three spatial components of electric field vector on one hand and electrostatic potential energy on the other. Due to the fact that the formal definition of electrostatic potential (V) entails unavoidable mathematical and abstract formalism, the physical picture of the concept of potential amongst UG students may be varied and therefore, the possibility of development of divergent mental models is high. Thus in the present work, we have investigated the mental models of the concept of electrostatic potential of a group of UG students majoring in physics.

RESEARCH QUESTIONS

- Does students face difficulty in understanding the concept of 'potential' in electrostatics? If yes, what could they possibly be.
- Do they consider that the presence of an electrostatic field results in existence of potential?
- Is it likely that they borrow idea from other concepts (such as potential energy, electric field) to associate a meaning to electrostatic potential?
- Is their mathematical understanding of 'potential' coherent with physical concept of potential?

METHODOLOGY

This research focuses on patterns of errors learners commit while solving the problems based on relationship between electrostatic field and electrostatic potential in the case of uniform electric field. The aim is further to explore if these errors are simple mistakes or do they have roots in students' conceptions of the scientific concept. Thus this study is situated in the qualitative paradigm where objective is to unravel the students' mental models of the concept as they solve problems. To this end we have selected three relevant questions from a standardized tool BEMA (Ding,Chabay,Sherwood & Beichner, 2006) . These questions were administered to the participants as a Quiz before the concept of 'electrostatic potential' was taught in the class. The questions were objective type with choices (a to g). For our purpose we had added another choice 'h' (any other). Students were required to choose the correct option in their view and also provide an explanation for their choice. Also after the Quiz each student was interviewed in detail about their responses and explanations. After the topic was done in the class another Quiz was administered with two multiple choice objective type questions. These questions were constructed by us and have 5 options (a-e) in which option 'any other' is included (Appendix). Again students were required to provide a correct choice followed by written justification for their choice and subsequently interviews regarding their explanations and choices were conducted.

The analysis has been done within the paradigm of constructivism with the basic assumption that each student constructs his/her own knowledge. The written responses and interview data was analysed together and some significant errors which were repeated by learners and had roots in students' conceptual understanding have been referred to as students' alternative conceptions and presented in the next section .

Participants for this research were a class of 18, 1st-year UG students of physics, studying at National Institute of Science Education and Research (NISER) situated in Odisha , India. The tests were conducted during the course on Electricity and Magnetism (EM). This is the first course of EM for students at undergraduate level, thus their prior conceptions about Electrostatics are those which they carried from their school level. Also, they have credited a course on classical mechanics prior to this course on EM.

RESULTS

The selected questions from BEMA are referred to here as Q1, Q2 and Q3. They are based on a situation where a uniform electric field is given in a plane and 4 points are considered. Students are required to find the potential difference between two points situated along the electric field in Q1, two points situated perpendicular to electric field in Q2 and two points which are diagonal to electric field in Q3. Options 'a', 'b','c' and 'd' of the MCQ indicate the potential difference between two given points as product of electric field and shortest distance between the given points with negative and positive signs . Choices 'e' and 'f' indicate the potential difference to be path dependent showing it to be the product of electric field and actual distance travelled to reach final point from initial point . Choice 'g' indicates the potential difference to be zero and another choice 'h' added by us indicates 'none of the above'. Other two questions constructed by the researchers are referred to as Q4 and Q5 and are given in Appendix. Students' choices are presented in table 1 given below. Students are referred to as S1, S2, S3…..S18 in the text.

Choices ---------- Question	a	b	c	d	e	f	g	h	Not Answered
Q1	S6,S14	S1,S2,S3,S4, S7,S8,S9,S10, S11,S12,S13,S15S16,S17,S18							S5
Q2							S1,S2,S3,S4, S6,S7, S8,S9,S10,S11, S12,S13,S14,S15, S16,S17,S18		S5
Q3	S6,S14	S1,S3,S7,S8,S9,S10,S11,S12, S13S15,S16,S17,S18				S2,S4			S5
Q4	S1,S2, S3, S5, S6, S7, S8, S9, S10, S11, S12, S13, S15, S16, S17, S18					-	-	-	S4,S14
Q5	S1, S3, S5, S6, S7, S11, S15, S17, S18			S8,S9, S10,S12, S13,S16		-	-	-	S2,S4,S14

Table 1: Students' Responses

We have observed the following alternative conceptions in the study.

E1: Potential is a vector

The concept of potential was specifically devised in electrostatics for the purpose of simplifying the complex vector calculations. It is negative gradient of electric field and thus a scalar quantity. While teaching, it is generally the third significant concept presented in the class after force and field. An important point to note is that force as well as field is a vector quantity. It was observed that many learners carried a strong alternative conception that potential was a vector. One of the reasons could be extrapolation of the properties of force and field which were presented prior in sequence to potential. This alternative conception was shown in many ways by learners specifically in Q1, Q2 and Q3. S2 directly expressed his belief in the interview that potential is a vector quantity. Further he could state the mathematical formula for potential to be a dot product between $\vec{E}$ and $d\vec{l}$, but he did not seem to understand the meaning of dot product as well (which is a multiplication between two vectors resulting in a scalar). On the other hand S17 depicted potential with a vector sign ($\overrightarrow{V}$) while solving the task in written form. He also depicted the relation between potential at two points in Q1 as $|V_1| > |V_2|$. Now, since the potential is a scalar quantity, the use of modulus does not hold any significant meaning here. While answering Q2, S17 mentioned "*potential is perpendicular to electric field*" i.e. he expressed potential in terms of a particular direction. It indicates that the learner extrapolated his conceptions of vectors to potential as well.

E2: Potential has an absolute value

Potential is defined with respect to a *conveniently* chosen reference, therefore we always measure potential difference instead of 'absolute potential'. While answering Q1, S17 wrote "*let* V1 = -Ew; V2 = -E +) = 0; V_2 -V_1 = –Ew; $|V_1| > |V_2|$ *which implies* $V_2 > V_1 = -ve$". Here, he equated potential at point 1 to '–Ew' which was incorrect since '–Ew' is the potential difference between point 1 and 2 rather than an absolute value at point 2. Also, it showed that he did not view potential to be a line integral of electric field over a path from one point to another. Instead, he understood potential to be having absolute values at various points in space which showed a clear alternative conception in his metal model.

Another manifestation of considering potential to have an absolute value was evident when S2 while answering Q4 marked the correct option but explained the reason in the following manner:

$$"V = -\int_a^b \vec{E}.d\vec{l} = -\int_a^b Ex.d\vec{l}\,, x, y = 0; \cos 90^\circ = 0;$$

At B and D V=0"; Since, points B and D were located on an equipotential surface thus, the potential difference between them was zero and not the value of potential itself.

E3: Only magnitude of field is important to calculate potential

Electric field is a vector quantity which comprises of both – magnitude and direction. While calculating potential difference between two points, dot product of field with distance between those two points is considered. It is important to note that the dot product involves only vectors and results in a scalar. Also, direction of both the vectors involved plays a crucial role while calculating dot product. Some learners were observed to believe that since potential is a scalar therefore to calculate potential only magnitude of field is important and the direction could be ignored which is an alternative conception. As a manifestation of this erroneous conception, S11 expressed in the first interview, that potential difference between any two points should be zero when magnitude of electric field is the same. She further added that these two points could be inferred to be on an equipotential plane, which is an incorrect inference.

E4: Negative sign in potential

Potential at a point P is defined as the negative integral of electric field with respect to distance i.e. . This negative sign depicts that the 'work', in bringing a test charge from infinity to point P, is done against the existing field due to some charges already present in the system. Thus, there is a physical reason for the negative sign being present in the equation. It was observed that students frequently missed this negative sign while calculating potential. Some learners were also observed to use it at their convenience, i.e. they used it in some questions and did not use it in other questions. Within the same task, S1 used it in one step and missed it in the other. Another student was observed to use a positive sign throughout the solution but brought a negative sign in the last step without stating any logic for inserting it. Due to the inconsistent use of the negative sign, at times learners were also observed to reach incorrect answers. For instance S6 got both his answers of Q1 and Q3 incorrect as he did not find any importance of negative sign in calculating potential. S2 although marked correct option as an answer to Q1, however while explaining in the interview he changed his answer to option 'a' explicitly mentioning that "*negative sign is only for indicating work done and not significant in calculating potential*". It shows that these learners lacked understanding of physical meaning of the concept as well as self-consistent mathematical framework to arrive at physically acceptable results.

E5: Limits of integration

Within a discipline, conventions are defined to make communication easier and less ambiguous. It helps everyone understand the same meaning of the given relation. In the case of integration, limits '*a*' and '*b*' are shown as $\int_a^b f(x)\,dx$ where '*a*' is the lower limit and '*b*' is the upper limit. Physically it means '*a*' is the initial point and '*b*' is the final point. While solving the integral the final relation is calculated as:

$$\int_a^b f(x)\,dx = [g(x)]_a^b = g(b) - g(a)$$

Learners were observed to show many errors regarding limits. Some learners (S1,S2,S4,S6) did not pay heed to the order of the limits (i.e. lower and upper) and calculated

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This interchange of limits changes the result by a negative sign which learners do not mention, resulting in incorrect answers, for instance Q1 and Q3 of S14. For some learners like S6 the order of limits did not matter while explaining the concept physically, he explicitly mentioned it in the interview. They interchangeably used the limits which showed that in their mental model the significance of order of limits as well as physical implications of a 'negative' sign in mathematical result was not important. Many of them felt that changing the sign would not make any difference because magnitude still remains the same. Whereas in the context of potential a positive or a negative sign represents whether work is done by the system or on the system which depict two independent and different cases. Another error noticed frequently was not mentioning the limits at all. Potential is a quantity which has a meaning only with respect to a reference point or in other words when a charge is moved over a distance (represented by dl) in the presence of electric field, only under that condition potential is defined. Hence understanding and mentioning the limits are very important while using potential.

E6: Potential is path dependent- Potential is derived from electric field using one of the most important properties of electric field i.e. conservative nature of field. It means curl of the electric field is zero i.e. it is irrotational. In other words, since curl of field is zero it can be expressed as a gradient of a scalar quantity which is defined as potential (V) in electrostatics. It also means that line integral of electric field is path independent or depends only on the end points. Therefore, in the relation

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot \vec{dl}$$

dl can be chosen to be any arbitrary path from *a* to *b*. That is, for each different path chosen *V(b) - V(a)* will remain the same, because it only depends upon the end points and not on the path. Some students tend to overlook this basic property while solving tasks. S4 marked correct option for Q1 and Q2 , however for Q3 he marked option 'f' which was incorrect. In the explanation to his choice, he substituted the value of dl to be $\sqrt{h^2 + w^2}$, which was the length of shortest 'distance' between point 1 and point 4. He perhaps could not appreciate the path independent nature of potential difference, due to which he could not combine the inferences from his answers in item no. Q1 and Q2. S2 on the other hand had chosen option 'f' for Q3 however in the interview he changed his option to 'a' (as he did for Q1).

E7: Field same means potential same- It was a common observation that students interchangeably used the names of the concepts e.g. field for potential or force for field. One of the manifestations of this error is seen when they extrapolate the properties of one concept into another concept. In response to Q2, the reason cited by S6 for his answer was *"since E is same for both points and that is why V should also be same"* or in other words 'if electric field at any two points is same then potential difference between those two points is zero' is an alternative conception. There could be conditions contradictory to this, such as given in Q1, where E is same for points 1 and 2 but still potential was different. In another instance, a student assumed that the 'electric field is constant' on equipotential surfaces, i.e. when potential is same field should be same indicating the alternative conception held by him.

E8: Gradient: Mathematical definition and physical understanding- 'Gradient' (of a scalar) is a mathematical operation which is used for determining the direction and magnitude of maximum change of a scalar. In order to mathematically estimate the 'gradient', the functional form of scalar function (whose gradient needs to be evaluated) should be necessarily known. However, if the scalar function is a constant over a surface, then it is expected that the 'maximum change' in that function will be along the direction perpendicular to the surface. In other words, the direction of the gradient of the scalar function will be along the direction perpendicular to the surface. Using this result, it is straightforward to show that the gradient of electrostatic potential will always be in a direction which is perpendicular to the equipotential surface. Many of the learners as shown in table 1 above could not ascertain this point and hence, they chose option d:'lack of information' in Q5. For instance, S10 mentioned *"there cannot be any electric field along the Y and Z direction anyway due to symmetry. To know the electric field, one needs to know V as a function of r, knowing it on a single plane is not sufficient"*. Although it is correct that one needs to electric electric field at two planes but the direction of electric field (negative of the gradient of potential) could be ascertained from the location of one equipotential surface as well. He missed this point which shows that he knows the mathematical aspects of a gradient of a scalar quantity but cannot connect it to a physical situation always.

DISCUSSION AND CONCLUSION

In the section above, following alternative conceptions have been observed: a) Potential is a vector b) Potential has an absolute value b) Only magnitude of field is important to calculate potential d) Negative sign in potential e) Limits f) Potential is path dependent g) Identical field amplitude implies identical potential (alternately, uniform electric in a region implies constant potential). Such an observations is made on the basis of repeated patterns which students exhibited in multiple situations during the interaction.

It is evident from the results that students' correct answer may not always project the real conceptual understanding as most students marked correct options for the given tasks. Their written explanations and oral interview gave us an insight into how they thought about the concept and how they linked one concept to another. It was observed a few times that the students wrote correct formulae however while using them to solve a problem, they got confused. The response of S6 in **E5** is a case in hand where the limits written by him (in written response) does not carry significant meaning. At times they could not substitute the variables in the formulae with the correct values given in the problem. A similar case is represented by S1 in **E4**. Incidents such as neglecting negative sign while calculating potential (S1 in **E4**), using incorrect order of limits or ignoring them altogether, not being able to utilise the concept of gradient to find direction of electric field (S10 in **E8**) shows a clear missing link between the mathematical representation and physical representation of the concept in their mental models. Further, aspects like interchangeably using the terms'field' and 'potential' at times (S6 in **E7**) not only shows naive error, it also reveals that the concepts are not delineated in students' mental models which, at times, result in one-to-one mapping of one concept to another. This issue may give rise to alternative conceptions such as considering potential as a vector (S2 and S17 in **E1**) and considering potential to be same at two points if electric field is uniform (S3 and S4 in **E6**). It, therefore, indicates a decent room for improvement in our teaching-learning classroom techniques in this topic which is accepted to be quite abstract. Possibly, after introducing the concept of 'potential' in formal manner, the students could be subjected to solve a variety of mathematical problems which require visualisation of concepts. For example, a question on estimation of 'field' and 'potential' for a symmetric charge distribution may be followed by tracing/drawing the equipotential surface. A well-designed mixed bag of questions related to one or related concepts may bring out inconsistencies in conceptual framework and subsequently rectify it as well. At times, a teacher needs to approach a problem in an unconventional (non-trivial) direction. Such reverse approach tends to create disequilibrium in case misconception/alternate conception exists. For example, estimation of a possible charge distribution from a pictorial representation of an equipotential surface could bring out inconsistencies in understanding of the relation between equipotential surface and electric field. However, this necessarily needs to be complemented with honest acknowledgement and analysis of the error. Eventually, such alternative conceptions must find an appropriate space in classroom discussions. Such an approach could help learners reflect on their concepts and find the points of divergence between what they understood and the widely accepted (well-negotiated) scientific concepts of the given time, further encouraging them to construct viable concepts.

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