LOGIC IN SCHOOL MATHEMATICS: THE OUTSIDER AT THE WINDOW

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The infinite we shall do right away. The finite may take a little longer.
— Stanislaw Ulam

Ulam might as well have been talking of the school mathematics curriculum, which inexorably leads the student to Calculus at its end. Infinite sets (like the set of natural numbers and integers) and infinite objects (like real numbers, rays) are pervasive in mathematics from middle school onwards, though in an intuitive rather than axiomatic form. *Finite mathematics* makes short, almost apologetic, appearances. A syllabus unit titled *Mathematical reasoning* is often included. Typically it is about propositional logic, and students are trained in verifying if a given boolean formula is a tautology. Since this part is allotted only about 4% of the teaching time in the whole year (of Class XI in India) with 2% weightage in the final examination, it is not taken very seriously by all concerned, who have $\sin \alpha + \sin \beta$, conic sections, the binomial theorem etc. to worry about, and they are surely more difficult. Mathematical modelling is largely absent from school syllabi.

Now, with the realisation that discrete mathematics lies at the foundation of computation, a demand for it is heard, with logic included in the package. *Computational thinking* is the new paradigm, but though this is about enumeration, repetitive patterns and discrete modelling, it is not (yet) considered to be a part of the mathematics curriculum.

Yet, all through school, students learn deductive procedures in equational theories and employ deliberate means of reasoning in algebra and geometry. Interestingly, the little logic introduced tends to be propositional logic rather than the logic of quantification, while the latter is the form of logic unconsciously used by the student in mathematics. Leaving this implicit has serious drawbacks, as for example evidenced by students asked to solve the equation: $\frac{1}{(x-1)} = \frac{x}{(x-1)}$.

Logic remains the outsider in the mathematics classroom, not far away but gazing in from the window, watching these plays.

Logic is not only about deductive reasoning. Logic is also a conscious use of formal language, understanding truth relative to models, figuring out consequence, relating assertions to algorithms that check those assertions, and studying limits to reasoning.

In this talk, we observe that all these are already implicit, scattered here and there, in school mathematics, and suggest that there is reason to explicate these, for curricular and pedagogic purposes, as well as to enrich teacher knowledge. We discuss how granting first class citizenship to logic in school mathematics can help with computational thinking as well.

Suggested Readings:

1. John T Baldwin, *Model theory and the philosophy of mathematical practice*, Cambridge University Press, 2019.

2. Viviane Durand-Guerrier, Logic and mathematical reasoning from a didactical point of view, Thematic Group 4, European research in mathematics education III. available from http://www.mathematik.unitdortmund.de/~erme/CERME3/Groups/TG4/TG4_Guerrier_cerme3.pdf

3. Herbert Enderton, *A mathematical introduction to logic*, 2nd edition, Academic Press, 2001.

4. Susanna S Epp, Proof issues with existential quantification, In *Proof and Proving in Mathematics Education: ICMI Study 19 Conference Proceedings*, F. L. Lin et al eds., National Taiwan Normal University, 2009.

5. Susanna S Epp, Logic and discrete mathematics in the schools, In *Discrete Mathematics in the Schools, DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, vol. 36.

6. Joseph G. Rosenstein, D. S. Franzblau, F. S. Roberts, Eds. *Providence*, RI: AMS Publications, 1997, 75-83.