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CONSIDERING A CONTRASTING CASES APPROACH TOWARDS DEVELOPING A RELATIONAL UNDERSTANDING OF THE EQUAL SIGN IN EARLY YEARS

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The reported study compared two instructional approaches for teaching elementary school students the relational meaning of the equal sign. The primary goal of the larger study, from which the data in this paper are reported, was to examine whether instruction that involves comparing the equal sign with contrasting relational symbols (the greater than– and the less than–signs) is more effective at fostering a relational view of the equal than instruction that focuses on the equal sign alone. Preliminary findings indicate the promise of using a contrasting cases instructional approach to promote appropriate learning about the ubiquitous equal sign in second grade.

THEORETICAL FRAMEWORKS AND SIGNIFICANCE

The theoretical underpinnings of this study are rooted in two areas of research in mathematics education: research focused on developing an appropriate understanding of the equal sign and research on contrasting cases approach to the teaching and learning of mathematics.

Understanding the equal sign in elementary school

Developing an understanding of mathematical equivalence is foundational to success in algebra (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015; Knuth, Stephen, McNeil, & Alibali, 2006). Unfortunately, years of research indicate that the equal sign remains largely misunderstood by young children (Behr, Erlwanger & Nichols, 1980; Carpenter, Franke, & Levi, 2003; Hattikudur & Alibali, 2010), with most elementary school students perceiving it as a request to compute (operational view) rather than a symbol indicating an equivalence relationship (relational view). For example, in equivalence problems with operation on both sides, such as 2 + 3 + 4 = __ + 4, children typically place either a 9 (adding addends on the left side) or 13 (adding all the addends on both the sides) in the blank. The relational understanding is essential not only for competence in arithmetic problems, but also for success with algebra (Carpenter, Franke, & Levi, 2003; Knuth, Stephens, McNeil, & Alibali, 2006), with an earlier developed understanding leading to better competence in algebra in later grades. It is also essential for success with non-standard arithmetic problems where the unknown appears in a place other than after the equal sign (e.g., 2 + _ = 4). As such, overcoming the misconception of the equal sign as an operational symbol early, before it becomes resistant to change (McNeil, 2014), is a crucial educational goal.

In fact, the Common Core Standard for Mathematics identifies that students should be able to interpret the meaning of the equal sign by the end of first grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Given this goal, the absence of explicit attention to the meaning of the equal sign in popular U.S. curriculum is concerning (Powell, 2012). The problem is exacerbated, as many preservice teacher textbooks offer little explanation about how to teach students about the equal sign (Li, Ding, Capraro, & Capraro, 2008). Furthermore, students are typically introduced to equal sign and symbols of inequality in the same grade. Thus, comparing and contrasting the equal sign to symbols of inequality may help in making sense of these symbols as relational in nature.

Contrasting cases to support mathematics learning

Researchers have used comparison of solution methods to promote understanding of abstract mathematical ideas (Rittle-Johnson & Star, 2007; 2011). While comparison of solution methods has been used for almost a decade in classrooms, research in recent years has evidenced promising effects of using comparison towards developing students' understanding of concepts underlying abstract mathematical symbols (Aqazade, Bofferding, and Farmer, 2016; Aqazade, Bofferding, and Farmer, 2017). For example, Aqazade, Bofferding, and Farmer (2017) found that second graders' who contrasted the cases of adding two positive integers and adding one negative and one positive integer were able to change their understanding of negatives in addition problems. With respect to the equal sign, there is evidence to support that contrasting the equal sign with symbols of inequality can help in shifting students' view of the equal sign from an operational to a relational symbol (Hattikudur & Alibali, 2010). The researchers found that third- and fourth- grade students who received instruction about comparing the three symbols made greater gains on conceptual tasks (that involved seeing the equal sign relationally) than those who received instruction on equal sign alone.

This study was motivated by the promising findings of Hattikudur and Alibali's work and was aimed at exploring the effects of using such an approach in early grades. The primary goal of the larger study was to examine whether comparing the equal sign with its contrasting symbols (the inequalities) is advantageous over explicit focus on the equal sign alone among first- and second-grade students who are arguably less entrenched in operational view of the equal sign (McNeil, 2014). In this report, preliminary findings on the effects of using comparison on second grade students' understanding of the equal sign as measured by their performances in solving equivalence problems is discussed.

RESEARCH QUESTION

Does instruction involving the use of a contrasting cases approach: specifically, comparing and contrasting the equal sign (as a symbol of quantitative equality), with the greater than- and the less than- symbols (as symbols of inequality), promote learning any differently than instruction without a contrasting cases approach in second-grade students?

METHOD

Participants

Participants (n=36) were recruited from two second-grade classrooms in one public elementary school serving a small mid-western community in the U.S. Participation was voluntary and required parent and student permission. Approximately 72% were eligible for free or reduced lunch and 49% were females.

Procedure

Students from each classroom were randomly assigned to one of the two conditions: (a) the *contrasting cases* group received instruction on comparing the equal sign with the greater than- and the less than- sign; and (b) the *equal sign only* group, that also acted as an active control and received instruction focused on using three separate definitions of the equal sign as a relational symbol. The number and type of problems seen by students in the two conditions were identical. Time spend on activities during sessions and any instruction, where applicable, were comparable across the conditions and were video recorded.

Students participated in a total of three one-on-one sessions. The first session was approximately 55 minutes long, and included: pretest, a short break, a lesson based on the group assignment with a discussion portion where students were introduced to the contrasting symbols or equal sign, and a session-exit test. During the second session, students discussed and identified how the contrasting symbols were similar and different, sorted a set of cards with number statements into a true or a false pile (e.g., sorting whether 2 + 5 = 1 + 6 belongs to the true or false pile) and reasoned why they identified the statement to be true or false. The third session was similar to the second session and ended with a posttest that included problems similar to pretest as well as transfer problems.

PRELIMINARY RESULTS

The outcome measures reported here consisted of students' performance on solving equations, in particular on solving equivalence problems with operations on both sides and one unknown, as well as the reasoning they used to arrive at their answer. Responses were coded for both correctness and the underlying way of reasoning the child used. The coding scheme for students' reasoning was based on a scheme used in prior research (Knuth et al., 2005; Hattikudur & Alibali, 2010). Out of 36 students, one did not complete all three sessions and was excluded from the study.

At pretest, with the exception of two students, all students got zero correct on equivalence problems. This is consistent with previous research (McNeil et al., 2012), which indicates that children typically fail on equivalence tasks, and this has been attributed to children having an operational view of the equal sign (McNeil & Alibali, 2005). Indeed, the majority of the students (98%) indicated an operational view, in their strategy choice by either combining the addends on the left side of the equal sign or adding all the addends when asked to describe how they figured out what the unknown on the pretest.

At posttest, Levene's test for homogeneity of variances revealed unequal variances and hence a one-way-Welch ANOVA was used. The analysis revealed that there were statistically significant differences in posttest scores across the instructional groups, Welch's F (2, 38. 31) = 4.79, p = .007. A Games-Howell post-hoc comparison (at $p < .05$) indicated that the students in the comparison condition (m = 4.68, s.d. = 3.67) performed marginally significantly different than students in the equal sign only condition (m = 3.10, s.d. = 3.32).

Identifying patterns in performance on equivalence problems

It must be noted that almost all students had zero correct on the pretest. At the posttest, the students ranged from getting some problems correct to getting all correct, with the bulk of students getting either all or all but one problem correct. A smaller number of students got some but not all problems correct, and thus present an interesting case. Initial interpretations of these students' performance suggest that these students might have only begun to consider the equal sign as something other than an operator but have not yet undergone a complete conceptual change in their understanding sufficient to demonstrate consistent performance on all equivalence problems.

To capture variations in student thinking and performance, students' overall performance was categorized into binary levels; with levels of learning defined as a) *satisfactory*, getting at least four problems correct (out of 6) with at-least one novel problem correct; b) *none*, getting none or just one of the posttest problems correct. These criteria allowed examining patterns of learning that may be associated with the different instructional conditions.

Overall, 62% of the participants' demonstrated *satisfactory* learning on the posttest and 21% demonstrated *no* learning. The percentage of participants by instructional condition with levels of learning is provided in Table 1. As can be seen, the percentage of satisfactory learners varied by instructional group, in the following descending order: *contrasting cases* group (54%), and *equals only* group (48%).

Differences in participants' learning as measured by number correct on the posttest was found to be related to the type of instructional condition, $\chi^2(2,36) = 12.15$, $p = .005$.

SUMMARY

The preliminary findings reported in this paper indicate that leveraging a contrasting cases approach to instruction might be useful in promoting a relational understanding of the equal with second graders and may help in circumventing the erroneous operational patterns commonly observed among young learners when solving equivalence problems. Further analysis will reveal how students progressed in their thinking and what aspects within the comparison seemed to bootstrap the notion of equal sign as a relational symbol among students on other equivalence tasks.

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