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# Cognitive and Affective Studies of STME

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# DEVELOPING 21ST CENTURY SKILLS AND STEM KNOWLEDGE IN PRE-SERVICE TEACHERS USING MAKERSPACE

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*This paper explores the use of a STEMinist Makerspace Project to promote engagement and learning of STEM content and support the development of 21st century competencies in female Indian pre-service teachers. Using a 'Makerspace Approach' these pre-service teachers participated in a series of activities: firstly, as 'students' creating their own artefact supported through a scaffolded approach by educators; then reflecting on these experiences and developing supporting questions; taking their artefact and the materials to school and using the questions to scaffold and support primary school students to create their own artefacts. Pre-service teachers reported on their increased engagement using the Makerspace approach and cited their development of 21st century competences, listing; collaboration, critical and creative thinking, problem solving and applying knowledge, as valuable to their own learning.*

## BACKGROUND

The Workshop, *Cross-Nation Capacity Building in Science, Technology, Engineering and Mathematics (STEM) Education* was held in the Regional Institute of Education (RIE), Bhopal, with female pre-service teachers. The pre-service teachers participated in three Makerspace-type STEM activities that provided them with opportunities to create and learn through practical experiences.

Fifty-two pre-service teachers (PST) participated in three Makerspace-type STEM activities that provided them with opportunities to create and learn through practical experiences. The focus of this paper is the evaluation of the STEM Makerspace programs, including the development of female pre-service teachers' skills, both personally and professionally, through the workshops and classroom activities. The research questions were:

1. How effective was the Makerspace Approach in supporting pre-service teachers' engagement in STEM education?

2. What 21st century competencies did the pre-service teachers identify and demonstrate as a consequence of their participation in the project?

## LITERATURE REVIEW

### A Makerspace Approach

Makerspaces are increasingly being heralded as opportunities for learners to engage in creative, higher-order problem solving through hands-on design, construction, and iteration (European Union, 2015). The Makerspace approach is different from a more traditional Makerspace. Traditional Makerspace has developed from a combination of online *Hackspace* (Copyright © 2016 London Hackspace Ltd.) or *FabLabs* (Copyright © 2015 Fab Foundation) and an actual physical place termed a *Place for Making* or *Makerspace* (Smith, Hielscher, Dickel, Soderberg, & van Oost, 2013). Makerspace sees artists and inventors coming together to create individual and collaborative original artefacts and can be anything from technology-rich items to knitting and craft materials.

The Makerspace approach sees makers situated in groups mentored to create a designated artefact. The artefact is presented to the makers who create and modify it to make it individualised and then take it home. This approach also has a definite and explicit focus upon the science, engineering and technology concepts involved, and the mentors are encouraged to use correct terminology as they question and support the school students (Blackley, Rahmawati, Fitriani, Sheffield, & Koul, 2018, p. 231). Table 2 below highlights some of the key differences identified in a targeted Makerspace learning activity.

| **Traditional Makerspace – recreational activity** | **Makerspace approach – targeted learning activity** |
|---|---|
| Makers create their own communities | Makers are organised into pre-determined communities |
| Makers choose materials at their own discretion | Makers are provided with a base-level kit of materials |
| Makers envisage and produce individual, often unique, artefacts | Makers are shown a completed base-level & operational (as appropriate) artefact and are challenged to construct a similar artefact |
| Makers are not mentored | Makers are mentored (not instructed) |
| Makers might evaluate their artefact | Makers are scaffolded to evaluate their artefact |
| Makers might be cognisant of underlying science, technology, engineering, mathematics or other concepts | Makers are made aware of related underlying science, technology, engineering, mathematics or other concepts in line with curriculum documents |

**Table 1:** Points of difference between traditional Makerspaces and the Makerspace approach

### Education in India

There are more than 1.5 million schools with over 260 million students enrolled and at a tertiary level it has about 864 universities, 40026 colleges and 11669 institutes that cater for 3.57 million tertiary students (Mattyasovszky, 2017; United Nations Development Programme [UNDP], 2015). Education is controlled by each state as well as centrally through the government in Delhi and each state has its own Board of Education controlled by the Central Board of Secondary Education (CBSE), and responsible for conducting exams for Classes X and XII. Each state has a State Council of Educational Research and Training (SCERT) while, for the country, there is the National Council of Educational Research and Training (NCERT) (Sharma &

Sharma, 2015) There are a number of STEM initiatives currently being implemented in Indian classrooms through a range of institutions and industry focused projects (Kishore Vaigyanik Protsahan Yojana, 2017, December 26).

**21st Century skills**

"Skills have become the global currency of 21st century economies" (OECD, 2012, para. 5), and therefore it is important to consider not only student knowledge but also students' skills. The globalisation and internationalisation of the economy along with the rapid development of Information and Communication Technologies (ICT) are continuously transforming the way in which we live, work, and learn (Voogt & Roblin, 2012, p. 299). The skills and abilities required will shift to more social and emotional skills and more advanced cognitive abilities such as logical reasoning and creativity (Author, 2019). (United Nations Educational Scientific and Cultural Organisation [UNESCO], 2007) suggests that education policies and curricula must aim to incorporate a broad range of skills and competencies necessary for learners to successfully navigate the changing global landscape and that the curriculum needs to ensure that students develop attributes and skills necessary for a rapidly changing society and workplace. Various terminologies are currently used to capture, compartmentalise and name this shifting cluster of competences, including *21st century skills* or *21st century learning* (Griffin & Care, 2014; Kids, 2015), *key competencies* (OECD, 2005), *soft skills*, *new collar jobs* (Bughin et al., 2018) and *entrepreneurial skills* (Foundation for Young Australians [FYA], 2015). The term *21st century skills* is widely used, but many argue that the skills and capabilities referred to were important well before the 21st century, while also noting that with rapid change, century-long milestones are inappropriate (Voogt & Roblin, 2012, p. 301). For the purpose of this research the term transversal competencies from UNESCO is adopted and this includes the range of skills encompassed in the categories in Table 2.

| UNESCO Transversal Competencies (2015) | 21stCentury Competencies (2008) |
|---|---|
| Inter-personal skills | Critical Thinking and Problem Solving |
| Critical and innovative thinking | Creativity and Innovation |
| Inter-personal skills | Communication, Collaboration |
| Global Citizenship | |

**Table 2:** Comparison of 21st Century Frameworks

**Research Design**

The methodology for this project was interpretivist qualitative research, based on an exploratory case study to examine pre-service teachers' engagement with and reflections on a Makerspace approach creating STEM artefacts – in this instance Wiggle bot, Catapult and Pipeline activities (Steminists, n.d.). The Wiggle bot artefact was a basic circuit, an upturned paper cup, with peg and pop stick (balance) with three pens as legs. Catapult required perseverance as PSTs received pop sticks, a plastic spoon and elastic band to create the model. PSTs were required to make the models from pictures and a video, without instructions, employing their theoretical knowledge and trial and error. The Pipeline was a team task where the group must build an enclosed pipeline over 2 metres long with 4 angles using only paper and tape which a small ball could travel down unaided. All the activities incorporated aspects of science, mathematics, engineering and technology. The research employed a paper-based survey of PSTs' engagement, including open-ended questions and observations to examine PSTs' engagement and reflections around their learning. The survey items and questions were developed and validated during previous international research (Sheffield & Blackley, 2016).

**Context**

The participant pre-service teachers (PSTs) were studying and living on campus at the Regional Institute of Education (RIE), Bhopal, a constituent unit of the National Council of Educational Research and Training (NCERT).

**Pre-Service Teachers**

Fifty-two female pre-service teachers studying in their 3rd and 5th semesters volunteered to participate in the workshops and were given STEMinist t-shirts and became part of the STEMinist community through the Facebook and website.

## METHOD

The Indian STEMinist program was implemented as shown in Table 3 below, with the phases following the Reflective STEMinist Identity Formation Model (Figure 1) that was developed from the Reflective Identity Formation Model (Authors, 2016). The phases for each activity were split, with phases one and two completed at RIE on the first two days in Bhopal, and then phases three and four completed at the Demonstration Multipurpose School.

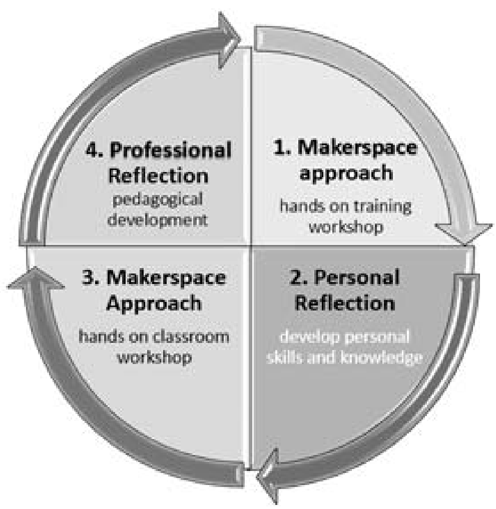


**Figure 1.** Reflective STEMinist Identity Formation Model

**Data Collection**

Anonymous surveys from 52 female pre-service teachers were collected as Part of Phase 2.

**Data Analysis**

The open-ended responses from the surveys were analysed using an aggregation of responses into themes;

including problem solving, creative and critical thinking, applied knowledge and collaboration. This was undertaken by two researchers independently and the results compared and moderated to ensure consistently. The group responses were analysed in the same way using the aggregation of responses into categories (Elliott & Timulak, 2005).

## RESULTS

After completing the activity workshops before working with primary students the pre-service teachers were asked about their enjoyment.

All the PSTs said that enjoyed the project and some offered multiple perspectives (as a consequence N = 79 from a participant group of 52) Table 3 shows their responses to questions

*What was the aspect that you most enjoyed?, What aspects did you find the most valuable?, What aspects did you find the most challenging?*

| Category | Responses (%) | | |
|---|---|---|---|
| | Enjoyable | Valuable | Challenging |
| Collaborating | 26 | 29 | 30 |
| Applying science & maths | 11 | 16 | 12 |
| Problem Solving | 14 | 23 | 19 |
| Hands on Activities | 20 | 7 | 12 |
| Pedagogical skills | 5 | 8 | 9 |
| Engagement | 13 | 2 | 9 |
| Creativity | 11 | 13 | 4 |
| Other | 0 | 2 | 4 |
| Total | 100 | 100 | 100 |

**Table 3:** PSTs' Responses

"I enjoyed, because it was teamwork, and also it was interesting, something which I haven't done before" was the response by one PST with 26% reporting that they enjoyed working with their peers. Another stated "It is the most creative learning project. It really teaches us to use the science, technology, engineering and mathematics in our daily life" with 20% of the PST listing a hands-on approach as a significant outcome. Twenty-nine percent of comments in this category related to collaboration and a further 23% to problem solving, with one PST summing up, "Problem solving and collaboration to solve a problem with cooperation and team is most valuable". PSTs were able to see the value of the activities in supporting their learning with a PST commenting, "Forcing us to think on our own. Motivating constructivism helps us to apply our knowledge and learn from ourselves".

Finally, PSTs documented the major challenges they experienced and 30% spoke specifically about issues with the Pipeline activity in including the number and size of the angles that were in the specifications and the use of the materials. They commented on the time constraints for all the activities and how having limited resources was also an issue.

Finally, in the PST were asked to consider the learning and make two comments about what they had learnt. These responses were aggregated and categorised into categories and these are presented in table 6. The majority of pre-service teachers, 66% articulated that they had developed a range of transversal competencies through the process of using the Makerspace Approach and creating artefacts. They reported that 32% developed collaborative and communication skills and were able to develop team skills. A smaller number 8% were focused on how the Makerspace workshops helped them to manage their time more effectively.

| Category | | Examples | % |
|---|---|---|---|
| General non specific | | I would love any such amazing and full of life opportunity once and many more times. | 6 |
| Time Management | | Keeping time limitation and unity | 8 |
| Pre-service teachers | | | |
| Self | General Knowledge | learn science technology and engineering | 4 |
| | General Skills | HOTS (High Order Thinking Skills) | 9 |
| | Communication | communicating our opinions to others in teams | 12 |
| | Collaboration | learning about communicating with others properly | 20 |
| | Problem solving | Thinking capability, problem solving skills | 9 |
| | Creativity | Creativity and collaboration | 6 |
| Teaching | General Teaching comments | Pedagogy- To teach learning by practical activity | 3 |
| | (School) Student learning focus | How to make the (school) students think (by asking questions) | 13 |
| | Total | | 100 |

**Table 4:** PST students to provide 2 new things that they have learnt related to your learning (N = 108 from a participant group of 52)

## CONCLUSION

*The Makerspace Approach in supporting pre-service teachers' engagement in STEM education*

The pre-service teachers engaged enthusiastically in the STEM projects and made the Wigglebot, Catapult and Pipeline in the hands-on workshop, they reported that they found the workshops helpful and were very engaged.

"It is the most creative learning project. It really teaches us to use the science, technology, engineering and mathematics in our daily life" a pre-service teacher reported. They found the hands-on tasks engaging and were able to articulate the STEM knowledge they had learnt in these relatively simple artefacts. The pre-service teachers all reported enjoying the project, however, some also articulated issues that were challenging. When Collaboration was focused on, a total of 20% of PSTs articulated that it was the most engaging aspect of the projects and then 29% reported it to be the most valuable aspect to the activities but then 12% reported it as the most challenging aspect and one that they often struggled to manage.

*Pre-service teachers identified a range of 21st century competencies as a consequence of their participation in the project*

The PSTs learnt that, through these engaging activities, learning can take place and creativity can be developed. They also expressed the opinion that the activities were a wonderful method for developing 21st century competencies also known as transversal competencies such as cooperation, reasoning, time management, problem solving, team work, precision, accepting defeat and rejection, thankfulness, collaboration, respect for others, listening and accepting others viewpoints, accepting what is useful and neglecting what is unwanted, concentrating even when facing failures and learning from mistakes and rectification. They could clearly see how these skills were important to their learning and how the skills were an important part of the Makerspace alongside the content knowledge. PSTs also articulated that patience, guidance, perseverance and a desire to genuinely help the students learn are key aspects that need to be developed for them to become effective teachers.

## ACKNOWLEDGEMENTS

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# EXPLORING THE USE OF DEDUCTIVE LOGIC IN GEOMETRY AS A TOOL FOR COGNITIVE GROWTH

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*This article presents some initial results from a study focused on promoting students' cognitive growth by building their deductive thinking skills in a college geometry course. The strategies used were ruler and geometric constructions and deductive proof.*

## INTRODUCTION

Historically, it is widely believed that learning mathematics fosters learners' cognitive development, and their problem-solving skills. Globally, mathematics is embedded within the school curriculum largely because mathematics is perceived as a subject that can help humans to learn to reason deductively, and apply such reasoning skills to everyday life. How to foster these reasoning skills? Most educators believe that deductive reasoning is a skill that must be acquired through careful and persistent nurturing. For students specializing in mathematics, by the college level, curricula include axiomatic reasoning and logic structures that are the foundations of mathematical proof, indicating that fostering cognitive growth by honing deductive skills is an important goal for mathematics education.

In this article, I present my work with a group of undergraduate students in a college geometry class. In this course, my goal was to intentionally foster students' deductive thinking skills by including tasks that would specifically target those skills. Thus, I intended to a) discuss the axiomatic development of geometric ideas so that students would appreciate the underlying structure, and use the structure to establish proof, b) include straightedge and compass constructions on a regular basis – students would do the constructions and prove that their constructions were correct, and c) encourage students to come up with their own proofs, with the intention of developing their deductive reasoning skills.

Specifically, the questions for my study were: (1) How can students be helped to appreciate and utilize the axiomatic development of Euclidean geometry? And (2) How can students' abilities in deductive logic be enhanced through construction and proving activities in geometry?

## THEORETICAL FRAMEWORK

The work presented in this article is based on a framework supported by research in two broad areas of mathematics education: a) research on cognitive development in mathematics in general, and on the cognitive development of proof, in particular, and b) research on geometric thinking. Specifically, I was interested in helping students to reason deductively and to understand and appreciate the basic underlying axiomatic structure of geometric thinking. In doing so, I hoped that students would gain cognitive skills in mathematics and more specifically, in geometry.

One of the basic premises of my work is that the two seemingly disjoint approaches in mathematics – empirical intuition and deductive logic – are two pillars on which cognitive development in mathematics rests, and it is the interplay between these two aspects that engenders learning. This viewpoint has led to research on how best to support the dialectic between these two areas of distinct yet complementary individual experiences. Students and professional mathematicians, each in their own way, rely on the constant interaction of both these strategies in order to advance mathematically. Lakatos (1977), an early proponent of this viewpoint, asserted that mathematical development is a result of the constant interplay of empirical observation and formal mathematics, even coining the term "abduction" to describe the synthesis. Clarifying this perspective, Schoenfeld (1986) pointed out that empirical knowledge and deductive knowledge are mutually reinforcing, and each enhances the other in significant ways. In particular, I believe that geometry offers an ideal vehicle in which to transact and observe the cognitive growth stimulated by the constant dialectic of empirical reasoning and deductive logic. There are several reasons for my belief.

First, in general, geometrical entities are easier to visualize and manipulate for students than other mathematical objects, and the access to visual representations facilitates the development of reasoning skills based on properties and attributes of geometrical concepts. Secondly, Euclid's approach to geometry, outlined in the *Elements*, was constructive. Many of Euclid's theorems are exercises in construction, based on deductive logic. Further, the constructions are accessible for students, most of whom are introduced to the basic constructions in school geometry. As students develop proofs for these results, they experience the element of problem solving. Thirdly, curricula in geometry have been quicker to evolve towards facilitating the interplay between the areas of empirical experience and deductive reasoning. Wirszup (1976) declared that (deductive) proof cannot be meaningful until the entities manipulated in the proof are meaningful. Thus, early on, educators broadly recognized that purely axiomatic initiation into the study of geometry is bound to be largely unsuccessful, and found ways to incorporate hands on experience with geometrical objects at the earlier stages of schooling via tangible objects, visual representations, and real-life experience. I believed that straightedge and compass constructions offer an excellent opportunity to combine these experiences and engage students.

The ground breaking work by Dina van Hiele-Geldof (1957) and Pierre van Hiele (1957) provided further support for a pedagogical approach to geometry that relies on enhancing students' experiences with geometrical entities, and reflecting on properties of the entities in an interactive manner. The van Hieles provided empirically based description of five stages of geometric learning that delineate the stages or levels that learners go through when developing ideas related to geometry. Modified from Lee, (2015), the levels may be articulated as follows:

- *Level 1*: Visualization: Students can recognize and classify shapes based on visual characteristics of the shape. They are unable to articulate properties of shapes.

- *Level 2*: Analysis: Students can identify some properties of shapes, and use appropriate vocabularies. They cannot use the properties for logical deduction.
- *Level 3*: Informal deduction: Students know the relationships among properties of geometric objects and are able to do informal logical reasoning. They cannot create formal proof.
- *Level 4*: Deduction: Students know the deductive systems of properties and can create formal proof.
- *Level 5*: Rigor: Students can do analysis of deductive systems and compare different axiom systems.

The van Hiele levels are hierarchical but non-discrete, and each subsequent level draws upon understanding built at the previous level. Students must gain sufficient experiences at one level before proceeding to the next, and students will function at different levels simultaneously, depending on the concept. In laying out these principles, the van Hiele model of geometric thinking is useful in two significant ways: 1) The model is useful in understanding where students are situated in terms of cognitive development, and has been used by researchers for this purpose, for example, by Mayberry (1983), 2) the model may be used to develop a pedagogical approach that helps students to transition from an earlier to a later stage, thus moving towards cognitive growth. In their study describing the van Hiele levels of geometric reasoning among students, Burger and Shaughnessy (1986) verify that the levels were useful in describing students' thinking processes on geometric tasks, and that the levels could be characterized operationally by student behavior. Their work suggests that the levels are useful in making pedagogical decisions about students' development in geometry, and designing tasks that aim to raising students to the next level.

The findings of the above research propelled me to use the van Hiele model to help me ascertain where my students were in terms of geometric thinking, and designing experiences that would help them to advance their geometric thinking to the next level. In their report, Tall et al (2012) traced the long-term cognitive development of mathematical proof. Their framework is initiated from perception and action, and evolves through proof by embodied actions and classifications, geometric proof and operational proof in arithmetic and algebra, to the formal set-theoretic definition and formal deduction. The research provided me with some pointers on how to design tasks that specifically foster cognitive growth.

## DETAILS OF THE STUDY

For my study, students were drawn from a college geometry class that I taught in the mathematics department of a state university in central New York. Many of the students were preservice secondary school teachers for whom the course was required; almost all of these students had mathematics as a second major. The other students in the class were mathematics majors who were taking the course as an elective. I encouraged the students to work in pairs or in small groups while working on construction problems or proofs. I collected data by observing student work, by writing detailed notes after class, and by taking pictures of student work.

## STRUCTURE OF THE COURSE

Most students in the class had taken a geometry course at the high school level. Based on their initial work in class, and on a homework assignment, I found that a majority of them were (mostly) at level 2 of the van Hiele model. They had understanding of properties and attributes of shapes, and tried to reason based on the properties. However, they reverted to the visual aspects of the shape when they encountered an obstruction in the problem-solving process, indicating a reversal to level 1 of the van Hiele model. A few of the students – about 3 or 4 – could be considered to be more at level 2. They tried to enunciate abstract definitions. Sometimes, they offered logical implications, based on the definitions. 1 out of the 22 students functioned mostly at level 3 of the van Hiele model. He was able to use deductive logic to create informal proof but often needed help with formal proof. He tried to incorporate new theorems into an existing network of geometric knowledge.

My objective in the course was to find ways to help students transition to level 3 of the model. This is the level where students think deductively, and I believe that if students became proficient at this level, then they would be sufficiently prepared for the courses that were to follow. To promote students' deductive skills, I chose largely two strategies (1) proof-based activities involving increasingly complex results that would include deductive logic, and (2) ruler-and-compass based constructions, that would include developing the construction, and justifying it. Both these activities would engage students in using the basic axioms and results of geometry, and would evolve as they added more results to their repertoire. Thus, I designed activities on an everyday basis that were geared towards these two strategies.

In the first few weeks of the semester, we laid out the basic foundations of Euclidean geometry contained in the first book of the *Elements* – beginning with the 23 definitions, the five common notions and five postulates, and the early propositions (http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html#cns). Among Euclid's first 12 propositions, seven involve construction. The students did these constructions, and for each one, students would prove the constructions were correct by using previous propositions and the common notions and postulates. A crucial component of the class was the small groups in which students worked – they pushed each other to justify their reasoning by asking questions such as "which result are you using (for a given step)?"

In addition, the textbook we were using (Libeskind, 2008) also facilitated the process. As in Euclid's *Elements*, the book formulated constructions as theorems requiring proof. For example, theorem 1.12 in chapter 1 characterizes an angle bisector as follows: A point is on the angle bisector of an angle if and only if it is equidistant from the sides of the angle. This theorem came up early in the course, and followed a section in which students defined and studied the properties of kites. Once students understood such a statement, they began to see that the statement reflected their construction, and was a tool to help them analyze their steps. Thus, students knew (from theorem 1.12) that in order to construct an angle bisector, they needed to construct the set of points that was equidistant from both arms of the angle. Having studied the properties of kites, some students knew that the diagonals of a kite are angle bisectors for the angles that they connect. They used this idea to construct a kite using (parts of) the two arms of the given angle (see figure 1).

Here, students used the result that the diagonal (BD) of a kite (DEBF) bisects the angles at the vertices that it connects. Thus, students drew a circle with center B, and radius BE. This construction created the sides

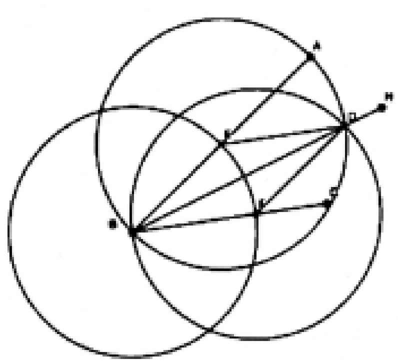

**Figure 1**

BD and BF of a kite. Students then completed the kite by drawing circles centered at D and F respectively with the same radius DE = FE (actually, students more often drew a rhombus). Thus, EB is the angle bisector of angle ABC.

Further, the exercises in the book required students to construct various geometric entities, and then prove that the construction was correct. The first few weeks helped to set the tone of the class, and to master the basic constructions.

Another important strategy that the book suggested for solving construction problems was outlined in three steps: Step 1) Assume that the construction is done 2) Analyze the construction and the shapes in it for various attributes and properties. Step 3) Use the properties and attributes to work backwards and carry out the construction. I strongly urged the students to use this strategy. An early attempt by students to utilize this strategy is described below.

**Episode 1**

Question from the exercise in Libeskind (2008, p. 38): A circle such that each side of a triangle is a tangent to it is called an inscribed circle. Assume that a tangent to a circle is perpendicular to the radius at the point of contact. Explain how you will find the inscribed circle, and then construct it. (Question restated in Figure 2)

Given: $\Delta$ MNO.
To construct Inscribed circle in $\Delta$ MNO

**Figure 2**

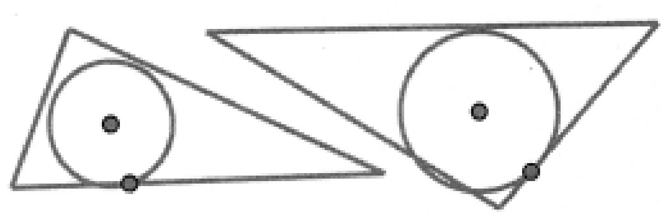

**Figure 3**

Students attempted this problem in pairs or in groups of three. Some students immediately drew a triangle and then tried to construct a circle inside it by guessing a center and a radius. Figure 3 shows the typical attempts. Clearly, this strategy did not work. I reminded them of the 3-step strategy outlined above. For step 1 of the process, students needed to assume that the shape had been constructed, and draw what the resulting construction would look like.

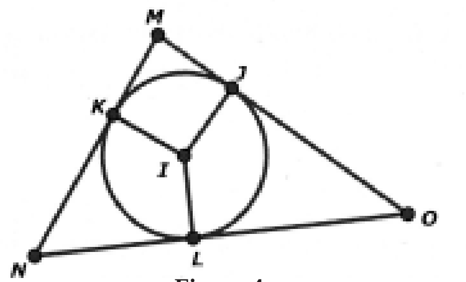


**Figure 4**

Soon, students realized that it was easier to draw the circle before they drew the triangle. One pair of students, Alexa and Joey, (pseudonyms), who were working together, drew the diagram in Figure 4 as a rough sketch of what the picture would look like if it were actually constructed. Thus, they first drew the circle (I), then chose three points (J, K, and L), then drew the three tangent lines. Then they proceeded to analyze the picture.

Their conversation was as follows:

Alexa: So, how do you think these (pointing to IJ, IK, IL) are related? I mean, I know they are radii but what else?

Joey: Yeah, there has to be something else, right? (They looked to me, and I stayed quiet)

Joey: Looks to me like these are perpendicular bisectors or something …

Alexa: You mean of the sides? Hmmm … yeah, but like, NL looks much shorter than OL. (After some thinking) So, did we draw this wrong? (Long pause)

Joey: But the sides have to be tangents, and they are, aren't they? (This last question was addressed to me, and I stayed quiet)

Joey: (after a pause) Well, the tangents have to touch at one point.

Alexa: (thinking) But that's not how we defined it. Look (turns back a few pages to show the highlighted definition in her notes). It says: the tangent is a line that is perpendicular to the radius at the point of contact. So, we have to assume that these lines are perpendicular to the radius.

After a while when I came back to them, they had marked the right angles IKN, ILO, and IJO.

**Discussion:** Analyzing this episode, I believed that Joey's purely visual reasoning indicated level 2 thinking – in the picture, he thought that the radii looked like bisectors (though even this was contradicted by Alexa). Similarly, their initial conception of a tangent being a line that "touches" a circle ("touching" being an undefined concept) was indicative of level 2 thinking. However, Alexa's reference to the definition as a step towards their construction indicated a readiness to move to deductive reasoning. I believe that reasoning on the basis of the definition was an important step in advancing students' geometric thinking, and showed a move towards level 3.

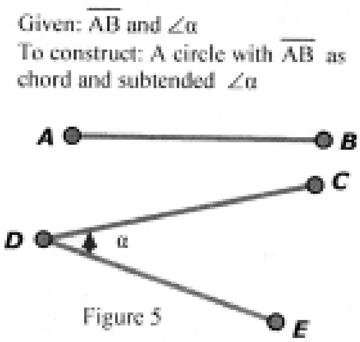


**Figure 5**

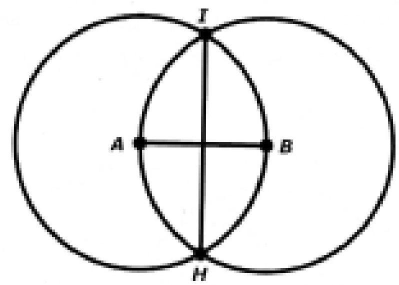


**Figure 6**

**Episode 2: A few weeks later, students worked on the following construction problem:**
Given an angle*a*, and a line segment *AB*, construct a circle that has *AB* as a chord with *a* being the angle subtended by *AB*. I asked students to draw their own angle *a,* and a chord *AB*.

When students worked on this problem (restated in Figure 5), they quickly pointed out that based on postulate 3, they needed the center and the radius of the circle in order to construct it. After some time, I joined a pair of students, Stephanie and Dana (pseudonyms), to observe their work.

Stephanie: Should we begin with the angle, or the chord?
Dana: So, if we could draw the angle, like this (she drew the vertex of *a* at the top of the page, and the arms extended downwards), then I can try to fit the chord *AB* (indicating line segments between the arms)? Right?
Stephanie: So you mean by *measuring*? (Her emphasis) Ha, ha! Not allowed! (They both looked at me, and laughed) Against the rules, right?
I: You've got that right! (We laughed together).
Dana: (thinking) OK. One thing I know. The center lies on the perpendicular bisector of a chord. We've proved and used that before – I remember that.
Stephanie: I agree. So, let's draw that. (When I came back a few minutes later, they had constructed a copy of AB, and constructed its perpendicular bisector IH – see figure 6). Now, how do we find the centre (of the circle) on this line (pointing on IH).

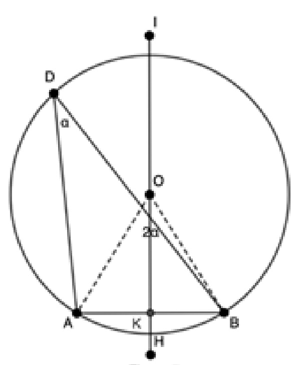


**Figure 7**

At this stage, they tried several unproductive strategies. Then, they overheard another student, Ben (pseudonym) talking about assuming that the figure had already been constructed. So, Stephanie and Dana decided to try that strategy, and drew the picture as in figure 7 (without the dotted lines). Then, they flipped the pages of their notebooks, considering various other results about circles that we had proven in class. Finally, they found the theorem that stated that for a given subtended angle on the circumference of the circle, the central angle subtended by the same chord is twice the subtended angle on the circumference (that we had done in a previous class).

| | |
|---|---|
| Stephanie: | (pointing to the diagram for the theorem) Doesn't this look like the picture we are trying to construct? |
| Dana: | Yeah, that's what I was thinking. We can do this (She drew the dotted segments OA, OB) |
| Stephanie: | Looks like we have two congruent triangles here. |
| Dana: | Where? |
| Stephanie: | (labelling K) See, right here. Triangle AKO, and triangle BKO. Because radii (OA, OB) are equal, AK = BK, and angles at K are right angles (pointing to each object). |
| Dana: | So, hypotenuse-leg (writing HL). OK, I see. So now angles at O (angles AOK, BOK) are equal to a. Hm, that is nice, isn't it? Because if we now make AD parallel to OH … |
| Stephanie | (interrupting) But AD is not parallel to OK. |
| Dana: | Yeah, but we can choose it that way because all these angles subtended on AB are equal, so we can pick one whose arm is parallel to OK. |

The students completed their construction using this idea.

**Discussion:** This was a challenging construction problem for students; they needed to use two theorems (that were relatively new to them) in conjunction. In order to combine these results they had to use deductive reasoning. The students would need to recognize and use the relationships between (a) the centre of a circle, and the perpendicular bisector of a chord and (b) between the angles subtended by the same chord at the centre and on the circumference. Students knew the statements of the related theorems but utilizing the statements to construct the centre of the circle forced them to apply the theorems in a new context Dana and Stephanie were able to use the relationship in (a) quite smoothly indicating level 3 thinking; however, the relationship in (b) above was a little more subtle. The crucial steps in recognizing and using that relationship came when they overheard Ben (thus being prompted by an outside source) and seeing the picture for the theorem. So, they were able to reason through the second part of the problem – a progression towards level 3 thinking; they even did some formal proving activities unprompted (recognizing congruent triangles and identifying the criterion) which is classified as level 4 thinking in the literature (Lee, 2015; Fuys, Geddes, and Tischler (1988)).

## CONCLUSION

At the beginning of the class, students presented their arguments citing properties of geometrical objects that they were very familiar with such as triangles, rectangles or parallelograms. We then discussed the initial part of the *Elements* in some detail, including some of the definitions, and why definitions were useful; in particular we discussed how the foundations of geometry as laid out by Euclid, were instrumental in moving learners forward in thinking deductively.

As evident from the two episodes described in this paper, there were marked differences in the ways that students built their constructions at the beginning and then towards the end of the semester. The students were definitely connecting their ideas more by using propositions and results that we had proved in class, indicating that they were ready to progress to a higher level of the van Hiele model. In the context of the model itself, it was evident that even for the same problem, (the same) students could function at more than one level simultaneously. As their instructor, this was exhilarating; as a researcher, this made the work of classifying the students by levels a little more thought provoking. The inter-student interaction was very helpful in maintaining a productive atmosphere in the class where students discussed ideas and defended their arguments with each other. Certainly, it gave me hope to extend my work and my efforts towards more work in this area.

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# DEFINITION OF AN ARTIFICIAL INTELLIGENCE ENGINE FOR MATHEMATICS EDUCATION

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*The National Education Policy Draft (2019) proposes that students across over a million schools have to achieve an adequate level of numeracy. While it may be a challenge to find quality teachers at such a large scale, certain facets of their wisdom and experience could be encapsulated in an Artificial Intelligence (AI) engine. The application of AI to education has been explored by various researchers. Here, we specifically investigate the considerations to develop an AI engine for school-level numeracy. Further, we incorporate these considerations into a probabilistic framework and propose the definition of such an AI engine.*

**Keywords:** *Artificial Intelligence, Mathematics Education, Probabilistic Framework*

## INTRODUCTION

Over the years, researchers have noticed that Artificial Intelligence (AI) engines can be trained to yield predictions that exceed human accuracy. This is achieved by applying a high quantum of computational effort to massive, and possibly ill-formed or poorly-defined data sets. Given this attractive feature, AI has been used for a variety of industrial applications (Nadimpalli, 2017). AI has been applied in education as well (Chassignol et. al., 2018), the aim being to have a program mimic the efforts of an expert teacher.

To build an engine capable of mimicking human effort, it is first necessary to understand how the learning process works, and how it is influenced by the teacher. At a middle-school level, an expert teacher may juggle limited resources to present different perspectives on a topic (Matic, 2019). At the secondary-school level, the teacher may need to respond to different understanding process used across students (Radmehr & Drake, 2019). In addition to methods of teaching and understanding, there are other factors that affect learning outcome. As pointed out in (Shah & Chandrashekaran, 2015), learning the sciences requires practice on the part of the student, and it could be beneficial to use the classroom as a practice session, rather than for just lecturing. Generations of teachers have educated students, and there is quite a challenge ahead to incorporate all of this perspective into an automated engine.

The focus of our effort has been to define a scalable education system for mathematics and numeracy that will help achieve the goals proposed by the Government of India’s National Educational Policy (Kasturirangan 2019). While it is not possible to have an expert teacher in every school in the country, it may be possible to scale a software-based solution. To succeed in this endeavour, it is imperative to sensitivize ourselves to the issues faced in urban, rural and tribal areas. A common obstacle encountered in both urban (Matthews, 2018) and tribal (Panda, 2006) areas is closing the loop with some assurance of expected learning outcome for the given funds. Moreover, social inequities also play a role (Harper, 2019), and should be handled with sensitivity. Fortunately, a software-based AI engine is agnostic to human aspects like socio-economic level, caste, or race, and could also go a long way in encapsulating the insights of experienced educators.

What we have accomplished so far is as follows. We have developed an Android-based application for practice of all aspects of mathematics from grade 1 to 10 of the CBSE syllabus. Well-trained teachers use this application to create customized sessions for each student with a combination of questions from various topics at selected levels of difficulty. Over 35,000 students have used this application, and a vast quantity of data has been generated. The next step that we would like to take is to automate the process of selecting customized sessions by quantifying the data collected, building a statistical model, and developing the required AI engine. This is the area of research of the current paper.

The current paper focuses on quantifying three aspects of the learning process, and then moves on to propose a design for an AI system. Firstly, we need a statististical model of difficulty level of each topic. This is addressed in section 2. Second, we need to characterize whether a student has exhibited proficiency in a topic from short-term memory or is the internalization of the topic deeper than that. Section 3 focuses on this aspect. Third, given that the various topics of mathematics have intricate interdependencies, to what extent would exposure to a related topic assist in mastering the current topic. This is addressed in section 4. Finally, in section 5, we use these empirical probabilities to construct a statistical framework to optimize the probability that a student will make progress. Various real-world constraints are included in the model, including the fact that mathematical topics often have prerequisites that would need to be first be mastered, and the fact that a class period is usually of limited duration, and this time would need to be optimally used. We now get into the details.

## STATISTICAL ESTIMATION OF DIFFICULTY OF A TOPIC

To estimate the level of difficulty of a topic, we not only need to measure the average marks in this topic, but also need to consider other factors including (i) quantifying the initial effort required to acquire a basic level of competence, (ii) the statistical variation of competence with stringency of requirement, and (iii) the variations in duration needed to attempt and succeed in various topics. Figure 1 sheds light on the first two considerations. The third consideration is addressed later in this section.

### Initial effort to acquire competence

The y-intercept of the graphs of Figure 1 indicate the number of attempts required to attain an intial level of competence in various topics. By initial level, we mean that the student should correctly answer at least 3 consecutive questions of this topic. The x-axis of this graph denotes the required number "n" of consecutive correct responses required.

Certain topics (like "Quadrilaterals") require a high level of initial effort to master the concept. In this topic, on an average, students have attempted 7.1 questions before they could correctly respond to 3 consecutive questions. On the other hand, in a topic like “Lines and Angles”, students, on an average, needed to attempt only 4.2 questions before they exhibited the same level of competence.

**Implication of Statistical Variation of Competence with Stringency**

It may be necessary to vary the stringency of the requirement “n” depending upon how often a topic is needed as a prerequisite. As an example, a topic like “Decimals” (taught initially in $5^{th}$ grade) serves as a prerequisite for a variety of topics in future years. However, a topic like “Bodmas” is a prerequisite much less often. For a topic like “Decimals”, the stringency requirement (i.e. the parameter “n” on the x-axis of Figure 1) may need to be increased.

Two features of the data stand out in Figure 1. The first is that on an average, some topics need more attempts to attain a given level of proficiency. For example, the graph for “Quadrilaterals” is higher than the graph for “Triangles”. In the ensemble of students tested, questions in the topic “Quadrilaterals” were more challenging than those in “Triangles”.

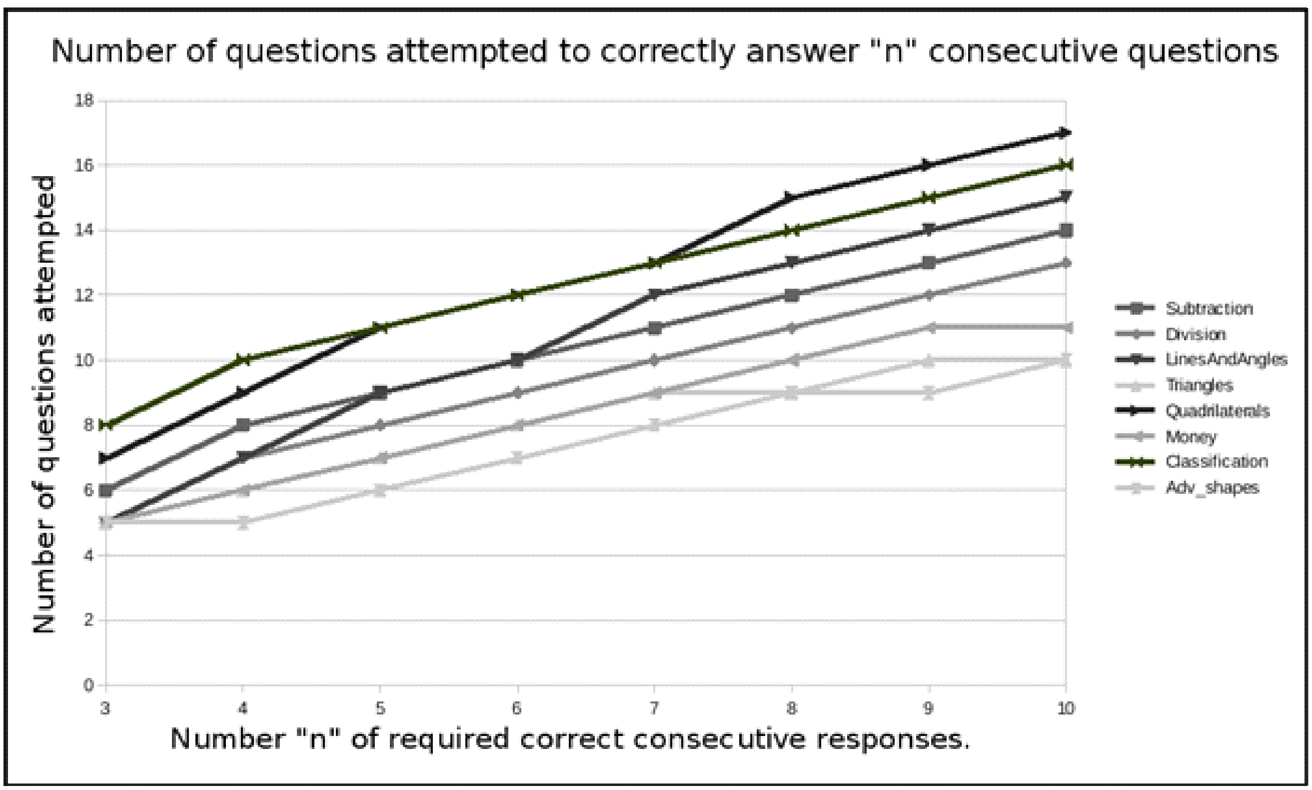


**Figure 1:** The number of questions required to internalize a topic increases with the parameter on the x-axis, and the variation is different for different topics

The second feature that stands out in the graph of Figure 1 is the shape of each curve, whether the second derivative is positive, zero, or negative. In some topics, the curve tends to have a positive second derivative, meaning it is accelerating upwards. The examples are “Quadrilaterals” and “Lines and Angles”. In these topics, after the student has gained some proficiency, there is a higher level of challenge to continue to maintain that proficiency. This often happens in questions which require a high level of focus.

In some topics such as “Division” and “Subtraction”, the second derivative is zero. Here, the level of focus required is just proportional to the number of questions given. While the duration required to solve a division question may go down with practice, the number of questions to be solved to meet a particular level of proficiency simply increases linearly with the required number “n” of consecutive correct responses.

Finally, some topics such as “Money” and “Advanced Shapes” exhibit a negative second derivative. Here, students may take a few attempts to determine how to solve the question. Once a level of proficiency has been attained, it is relatively easy to maintain that level.

**Average Percentage and Duration**

Given an ensemble of students, there is a variation of average percentage attained and mean duration required across topics. A particularly useful metric is the mean duration, because this is an indicator of how much effort the student is putting into this topic. These values are tabulated in Table 1.

In Table 1, we can see that topics can be grouped into three clusters: (i) Highly procedural topics like “Subtraction” and “Division”, (ii) topics that initially require thinking but for which the student eventually derives a procedure, like “Lines and Angles”, “Triangles”, “Quadrilaterals” and “Money”, and finally (iii) topics that are essentially logical reasoning (like “Word Classification” or “Advanced Shapes”) for which there is no standard procedure.

| Topic | Average Percentage | Mean Duration (Minutes) |
|---|---|---|
| Subtraction | 95.13 | 3.10 |
| Division | 86.19 | 2.64 |
| Lines and Angles | 84.65 | 1.17 |
| Triangles | 82.84 | 1.07 |
| Quadrilaterals | 80.34 | 0.73 |
| Money | 79.69 | 1.08 |
| Classification | 63.79 | 0.49 |
| Advanced Shapes | 86.32 | 0.21 |

**Table 1:** The average percentage attained and mean duration required while solving questions in various topics

In highly procedural topics like “Subtraction” and “Division”, students do take a long time to solve questions, but are not particularly flummoxed by the type of question or the juxtaposing of this question with questions from other topics (discussed in the next section and elucidated in Figure 2).

In newly introduced mathematical topics like “Lines and Angles”, the student usually requires a while to determine how such questions are solved, and eventually develops his/her own procedure to solve these questions.

When we have topics focusing on logical reasoning, like “Word Classification” or “Shapes”, the student is able to quickly answer the question, but the frequency of getting the answer correct may or may not be high. This does not necessarily imply that the student is guessing (although that continues to be a possibility), but could mean that determining the classification or matching shape is a quick process without the labor of a standardized procedure.

All of these metrics to measure the difficulty of a topic need to be modulated by an orthogonal factor, which is whether the student is attaining competence using short-term memory, or is truly internalizing a topic for recollection at a later period in time.

**Short-term versus Medium-term Proficiency**

There are various time-frames in which to measure the idea of competence, roughly speaking, the short-term, medium-term, and long term periods (Norris, 2017). A short-term approach would be having a student attempt only one particular type of question until s/he starts getting it correct over a number of consecutive attempts (for example, “Fractions”). This offers both an intellectual and psychological benefit to the student. Once this level of proficiency has been attained, the next stage (i.e. medium-term horizon) would be to solve a question in a given topic interspersed with questions from other topics. In the longer-term, the student would be expected to answer questions from this topic many months after first attempting it, for example, in a final exam. In this research, we focus on the short and medium term.

In Figure 2, we show the empirical probability of being posed questions from mixed topics while correctly solving questions in various topics. The larger this number, the more ability students have, on an average, to correctly solve questions in this topic while switching context. Based on whether a topic is highly procedure-based or application-based, the ability to switch topics tends to vary. The foundation for the metrics and selection process (choosing topics, number of questions, measuring consistency) is discussed in (Swaminathan 2015).

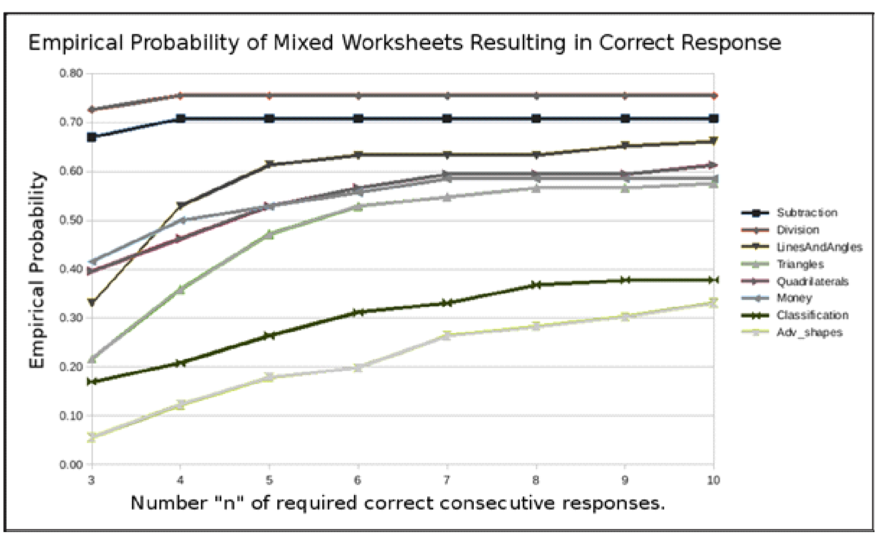


**Figure 2:** The empirical probability of allowing mixed topics while acquiring proficiency in any topic tends to increase as the parameter on the x-axis increases.

In highly procedural topics such as “Subtraction” and “Division” it is easy to gain proficiency and continue to maintain this proficiency despite the context being switched between different topics. Even with as high a context switch probability as 0.75 (i.e. 75% chance of context being switched), students, on an average, are able to attain and maintain proficiency.

Then, we have a topic like “Lines and Angles”, which starts off requiring a high level of single-topic focus (only 33% of the time does a switch in topic result in a correct response), but moving up to the possibility of a high level of multi-topic focus (can switch topics 67% of the time). Our teachers have observed that in such topics, the student needs single-topic focus to understand a new concept, and once this concept has been internalized, it is easier to handle this topic in conjunction with other topics.

Finally, we have topics in the realm of logical reasoning rather than conventional syllabus mathematics. Examples include “Advanced Shapes” (finding the matching shape) and “Classification” (word classification). In such topics, the data indicates that the student finds it difficult to retain proficiency in this topic while switching between topics.

The variation has dependence on whether the topic is highly procedural or more application-based. It is of interest to compare the data of Figure 2 to that in Table 1 since the clusters observed in both data are similar. In highly procedural topics such as “Subtraction” and “Division”, Figure 2 indicates that the student is able to handle the topic interspersed with other topics, and Table 1 indicates that the student requires a reasonably long time to perform this procedural task. In newly-introduced mathematical topics like “Lines and Angles”, “Triangles”, “Quadrilaterals” and “Money”, Figure 2 shows that the student initially needs to focus exclusively on this topic, but as competence is attained, is able to switch between other topics. Hand-in hand, Table 1 indicates that the student requires just about 1 minute to solve the question. The third cluster comprises topics focusing on logical reasoning, like “Word Classification” and “Advanced Shapes”. Figure 2 indicates that the student needs to focus exclusively on such topics to gain proficiency, and Table 1 indicates that the duration to solve these questions is quite low. There is no elaborate procedure to classify or select a matching shape, and questions can be answered quickly. However, it appears to be challenging to attempt such questions after answering questions of a completely different nature.

## INTER-TOPIC DEPENDENCIES

Mathematics is a highly interconnected science. The solution to a problem may be inspired by working on an area of mathematics that bears but a subtle relation to the original problem. This observation inclines us to ask the question, “In order to correctly solve a problem in the current topic, is there another topic that could be presented prior to this that would perhaps stimulate a style of thinking conducive to solving this particular problem?”

| **To get a question correct in Division** | | **To get a question correct in Lines And Angles** | | **To get a question correct in Quadrilaterals** | |
|---|---|---|---|---|---|
| Topic Given Earlier | Frequency | Topic Given Earlier | Frequency | Topic Given Earlier | Frequency |
| Division | 1203 | Lines & Angles | 1343 | Quadrilaterals | 790 |
| Multiplication | 331 | Area Volume | 290 | Area Volume | 102 |
| Place Value | 269 | Percentage | 42 | Circles | 59 |
| Subtraction | 95 | Decimals | 31 | Percentage | 20 |
| Fractions | 62 | Circles | 19 | Decimals | 13 |
| Mixed WP | 55 | Triangles | 15 | Lines & Angles | 12 |
| Factors Multiples | 33 | Fractions | 11 | Fractions | 9 |
| Measurements | 21 | Factor Multiples | 10 | Triangles | 9 |
| Percentage | 16 | Olymp Time | 7 | Directions | 8 |
| Area Volume | 16 | Word Classification | 6 | Mixed WP | 7 |

**Table 2:** Frequency of topics attempted prior to getting a problem correct in the current topic

One can shed light on this matter by examining, for any given topic, the frequency with which various topics were attempted prior to correctly answering a question in this current topic. As can be seen in Table 2, prior to getting a question in "Quadrilaterals" correct, students had attempted questions in the same topic (i.e. "Quadrilaterals") 790 times, and had attempted "Area Volume" 102 times. To be clear, the students have correctly solved questions in "Quadrilaterals" 102 times after merely attempting a question in "Area Volume", and not necessarily getting the question in "Area Volume" correct.

In all cases, the best prior topic appears to be the topic itself, but this observation may have to be taken with a nuanced viewpoint. Often, the teacher makes a student continue to work on a particular topic until s/he attains a reasonable level of competence. Even under such a possibly suboptimal circumstance, it is noteworthy that working on other topics may have a significant desirable impact on gaining proficiency in the current topic.

Given that an Artificial Intelligence engine can handle mammoth quantities of data and run algorithms that are based on probabilities, the solution that suggests itself is to use each topic as a prior with a frequency proportional to its empirical probability, and continuously update the table of empirical probabilities.

## DEFINITION OF AN ARTIFICIAL INTELLIGENCE ENGINE FOR MATHEMATICS EDUCATION

In this section, we define an Artificial Intelligence engine for mathematics education. The main objective of this section is to utilize the findings of the previous sections to create questions for the student that will be at the appropriate level and maximize the probability of the student making progress, given the time constraint of the class period.

We start by defining a few terms, then list out the empirically obtained quantities required. Then we build up the criterion that will be optimized to decide what question should be given to the student. Finally, we define the engine that uses a Bayesian model to generate a customized sequence of questions that will be optimal for the student.

Let us establish the terminology of “Level”, “Prerequisite Factor” and “Accessable Level”.

**- Level:** For each topic in mathematics (like “Addition”, “Fractions”, etc.), there would be several “Levels”. Roughly speaking, a Level is a section of a chapter of the textbook. So if the level is 503, that might refer to the 3rd section of a 5th grade textbook. We define $C_i$ to be the event that the student acquires consistency in the i-th level, and $t_i$ to be the expected value of duration to correctly answer questions at this level.

**- Prerequisite Factor:** Each level in mathematics could serve as a prerequisite to a variety of other levels. For example, a level like one-digit addition would probably serve as a prerequisite to most of mathematics in the current and future years. The “Prerequisite Factor” is a measure of how many future levels in various topics depend on this particular level. We define $F_i$ to be the prerequisite factor for the i-th level. Thus, $F_i$ for one-digit addition would be a high number.

**- Accessable Level:** A level for which all prerequisites have been successfully completed is said to be an “Accessable Level”. Let ***A*** be the set of all currently accessible levels. This will be set on a per-student basis. Now, for each level, the quantities required will be: (i) the required number “n” of questions to proficiency (the x-axis of Figures 1 and 2), which is decided by a panel of experts, (ii) the average number of questions to gain proficiency, which is obtained empirically, (iii) the average duration needed (obtained empirically), and (iv) the probability of having a mixed session (obtained empirically from Figure 2).

Using these quantities, we would like to define a criterion “J” to be optimized. Given a maximum time “T” and the constraint “M” that we want mixed topics if possible, we define

$$J=\sum_i F_i P\left(\frac{C_i}{M}\right) \text{such that } \sum_i t_i \leq T \text{ and } i \in A$$

as the criterion to be optimized. Essentially, we would like to maximize the expected value of prerequisite factors of competence, given that we are doing mixed topics and constrained to accessible levels, within the stipulated duration of a class period.

The value of $P(M/C_i)$ can be read from Figure 2, and emiprical values of $P(M)$ and $P(C_i)$ can be extracted from data. Using Bayes’ Theorem, we get

$$P(C_i/M) = P(M/C_i)\ P(C_i)\ /\ P(M)$$

which can be used in the criterion J to be optimized. Further, the specific set of mixed topics can be selected probabilistically, by creating a probability density from the data of Table 2.

The levels selected will be determined by a greedy search algorithm, maximizing the values of $F_i$ with $\notin \boldsymbol{A}$. Also, $\boldsymbol{A}$ is updated each time the required proficiency of any level is attained.

The above algorithm maximizes the expected value of prerequisite factors of competence using empirical probabilities from a global ensemble. Future work includes modifying these probabilities on a per-individual basis, and a multi-resolution approach for prerequisites.

## CONCLUSION

It is a signifcant challenge to enable all of the schools of India (over a million in number) to have the benefit of an expert teacher for mathematics. However, some facets of such expertise could be codified into an Artifical Intelligence engine, and scaled across schools. The National Education Policy Draft (Kasturirangan, 2019) specifically mentions technology as an enabler of quality education at scale. The current paper has outlined the issues related to and the process of developing an AI engine specifically for mathematics education.

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# LEARNING CONTROL SYSTEM DESIGN USING NANO DRONE IN A PBL FOCUSED ONLINE ROBOTICS COMPETITION

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*Teaching advanced conceptual knowledge and practical skills in a hands-on-manner to a large number of students is a challenge. The e-Yantra project hosted in IIT-Bombay, through it's e-Yantra Robotics Competition (eYRC) for college students, teaches these skills scalably. Participation is free and hardware is shipped to participants who are mentored constantly throughout the competition. The 7th edition of the competition, eYRC-2018, had a theme (a gamified problem statement), called "Hungry Bird," that taught Marker-Based Localization, Path planning using OMPL, and Waypoint Navigation using PID on a nano-drone using open source platforms such as ROS and V-REP. This paper outlines how we optimally designed and deployed these concepts as a series of tasks which eventually helped us to quantify the learning outcomes among students. 832 students were assigned this theme, and to achieve scale most of the tasks were automatically evaluated. Finally we illustrated how we have achieved the effectiveness of the theme with task results and participant's feedback. This study and its outcomes are beneficial for academicians seeking to teach advanced engineering skills at a large scale.*

## INTRODUCTION

e-Yantra, a project at IIT Bombay funded by the Ministry of Human Resource Development (MHRD), exists to develop and deploy transformative digital pedagogies to train both college faculty and students the concepts of robotics and embedded systems. One way e-Yantra implements this for students is through the e-Yantra Robotics Competition (eYRC) (Krithivasan et al., 2014a) that has been growing exponentially since it's launch in 2013 having had 28000 registrations (as teams of four students) in the 2018 (eYRC-2018) edition. The eYRC competition has effectively shown that students learn while competing and compete while learning (Krithivasan et al., 2014b) and delivers hands-on learning to a large number of students across the country at the undergraduate level. The training follows a project-based learning approach that teaches participants core skills in robotics and embedded systems by having them solve problem statements that are gamified instances of real world problems termed "themes." A theme contains a series of tasks culminating in a final implementation of a theme with continuous mentoring and progress evaluation. Using the model, students were successfully trained in Image Processing (Krithivasan et al., 2016), 3D modelling and Designing (Karia, 2018) and much more.

Following the same model, a complex theme entitled **Hungry Bird** was introduced in eYRC 2018 which aims to teach participants a general understanding of control system design by Waypoint navigation using parallel PID controllers on an external control loop based on marker-based localization to command velocity of a nano-drone in reference to its pitch, roll, yaw and throttle. Participants were taught marker-based localization using WhyCon (Nitsche, Krajnik, Cizek, Mejail, & Duckett, (2015) and ArUco (Babinec, Jurišica, Hubinský, & Duchoò, (2014) markers, Computing global path using OMPL (Sucan, 2012), Waypoint navigation using PID controller (Åström, 1995) and methods to tune PID parameters. The competition also aims to give students exposure towards Scripting languages such as Python and Lua, open source simulation platform V-REP (Rohmer, 2013) and middleware, ROS (Quigley et al., 2009). This theme was also an experiment in auto evaluating submissions of a large number of students. This paper focuses on the design aspects of tasks and learning outcome of this theme.

This theme showcased the diligence of parent birds by conveying a story of a bird feeding its young. A drone represented the bird whose job was to autonomously fly through a series of (hula) hoops to signify gathering and subsequently feeding fledglings. This paper describes how we designed and deployed these concepts as a series of tasks. Each task aims to impart certain skills which helps the participants to solve the Final problem statement.

28,976 students registered for the competition in teams of 4 thus amounting to 7244 teams from colleges across India, Nepal and Bhutan. The teams have to first pass a selection test which tests them for knowledge of basic electronics, basic programming and aptitude. Following this shortlisting criteria, 1544 teams qualified to participate in the competition out of which 208 teams were assigned this theme.

## COMPETITION DEPLOYMENT AND INSIGHTS GAINED

The Hungry Bird theme was conducted in two stages as illustrated in Figure 1.

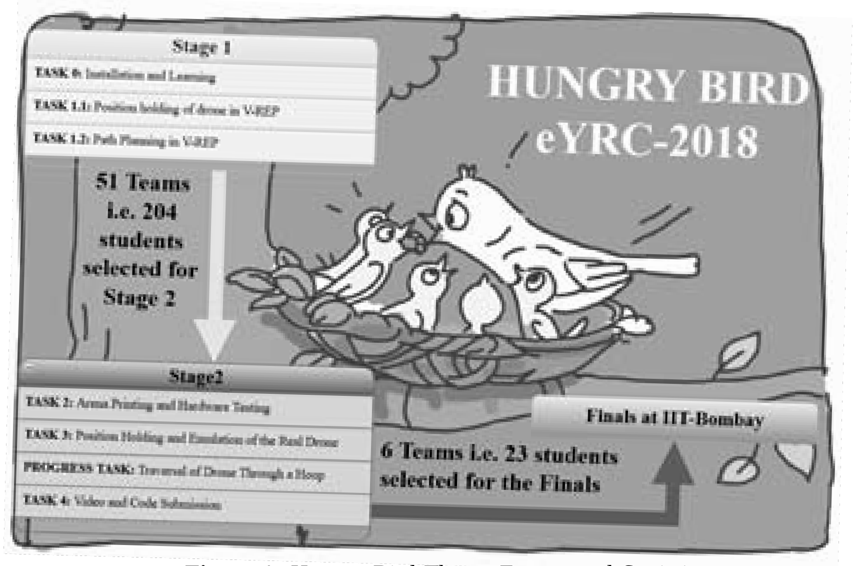


**Figure 1:** Hungry Bird Theme Format and Statistics

**Stage 1 : Participants compete using a drone model in a Simulator**

Stage 1 consists of two tasks, viz., Installation and Learn (Task 0) and Implementation of the theme in the simulator, V-REP (Task 1).

**Task 0 : Installation and Learn**

This task aims to introduce participants to Robot Operating System (ROS) and V-REP (Virtual Robotics Experimentation Platform). Participants are taught about the Marker Based Localization system using Whycon and ArUco markers. Tutorials to learn Linux, Python, Basics of ROS and V-REP[1] and ROS packages[2] for interfacing V-REP and ROS, WhyCon and ArUco markers were provided to the teams. The task involves computing position of WhyCon and ArUco markers using the feedback from an overhead vision sensor in V-REP. The task was graded in a binary way if they have successfully displayed the output or not.

**Figure 2:** Drone model in V-REP

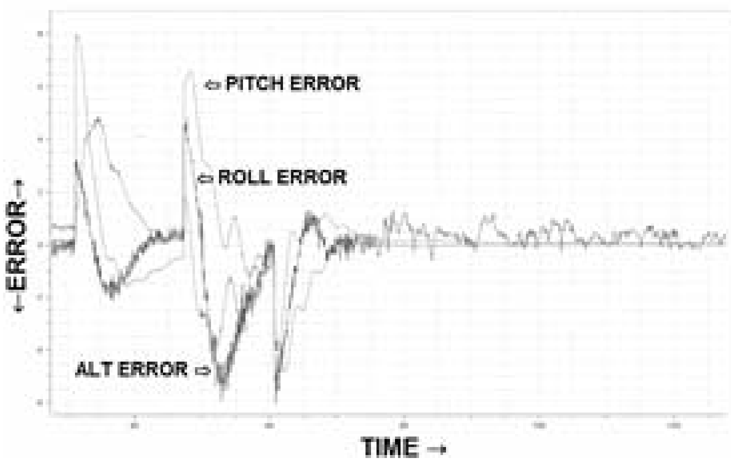


**Figure 3:** Plotting errors for debugging

**Task 1 : Implementation of the theme in the simulator, V-REP**

Task 1 consisted of two sub-tasks, viz., Position holding of drone in V-REP (Task 1.1) and Path planning in V-REP (Task 1.2).

**Task 1.1 : Position Holding of Drone in V-REP**

This task aims to teach participants about PID control by implementing algorithms in V-REP. A well designed drone model named “eDrone” as shown in Figure 2, which responds to ROS commands, is provided to the team. The task involves implementation of a PID algorithm to control the drone via ROS commands to hold its position at a given setpoint using the Localization system which the participants have learned in Task 0. Video tutorials[3] were provided to learn how to implement the control algorithm and how to effectively tune the PID parameters. Classical PID algorithm and ways to improve the algorithm to the task requirement are

1 https://youtu.be/l5RDBuIM3U8, https://youtu.be/ioNNvy805-4, https://youtu.be/DylQbGCF5ps, https://youtu.be/WB0zCufrHOM

2 https://github.com/fayyazpocker/vrep_ros_interface, https://github.com/lrse/whycon, https://github.com/pal-robotics/aruco_ros

3 https://youtu.be/BJ-hkJ2kdR4, https://youtu.be/7KcMoazeeTM, https://youtu.be/SNO_Vm7bpio

also taught. They are taught how to interpret the effectiveness of the algorithm using real time plotting of errors as shown in Figure 3**.** Use of Ziegler-Nichols (Ziegler, 1942) method to tune PID parameters and in-flight tuning of PID parameters using ROS topics are also taught.

Teams were instructed to submit a log file (rosbag) which contains information of the pose of the drone throughout the run. The log file was automatically evaluated based on an algorithm which takes the parameters like time taken by the drone to reach the setpoint, overshoot of the drone from setpoint and stability of the drone at setpoint. Code submitted by the teams were evaluated based on computation of error, Proportional, Derivative and Integral Term in PID and Sampling time of PID. This task comprised 40 marks of which 20 marks were based on automatic evaluation of the rosbag file and 20 marks were based on evaluation of code.

**Task 1.2 : Path planning in V-REP**

This task helps participants learn to compute global path using OMPL plugin available in V-REP. A V-REP scene as shown in Figure 4 with obstacles and target locations are provided to teams. The task is to control the drone to reach each target point represented by blue, green and red spheres whilst avoiding obstacles. Required video tutorials[4] for the completion of the task were provided. Learnings in Task 0 and Task 1.1 will help in completing this task. Similar to Task 1.1, the log file submitted by the participants were used for automatic evaluation of the submission considering parameters like time taken by the drone to cover all targets, the number of targets the drone covered and the deviation from the optimal path. This task comprised of 60 marks out of which 40 marks were based on automatic evaluation of rosbag file and 20 marks for code evaluation. Cumulative marks of Task 1.1 and Task 1.2 were considered for selection for Stage 2. Figure 5 shows mark distribution graph for Stage 1 of 208 teams. The cut off for selection to Stage 2 was set to 50 marks. Out of 208 teams, 51 teams got selected.

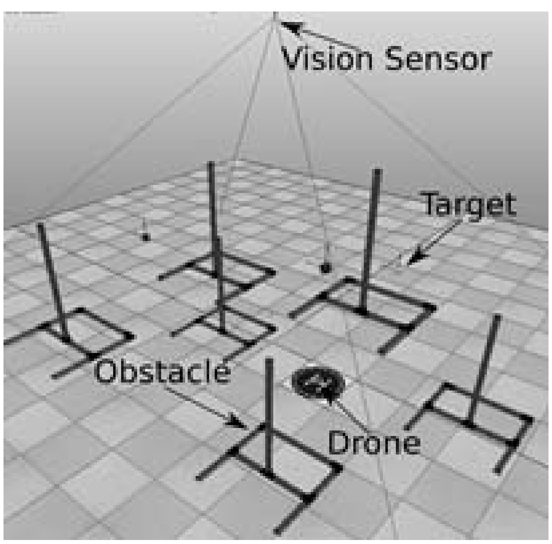


**Figure 4:** V-REP scene for Task 1.2

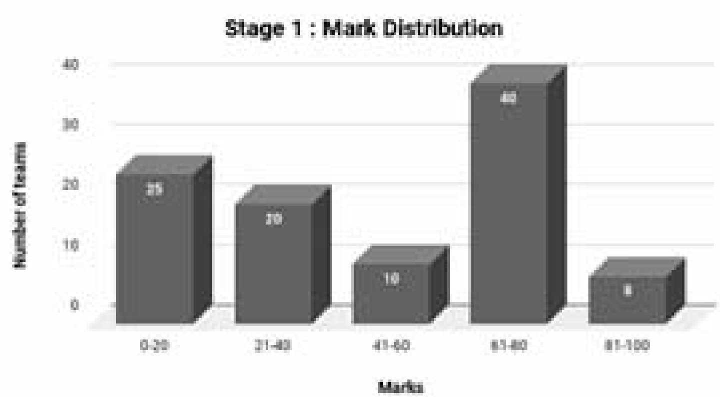


**Figure 5:** Mark distribution in Stage 1

[4] https://youtu.be/F9U-cCAoBM8, https://youtu.be/beHLO-E6bgI

**Stage 2 : Participants compete with given drone kit**

Stage 2 consisted of four tasks, viz., Arena Printing and hardware testing (Task 2), Position holding and emulation of real drone (Task 3), Traversal of drone through a hoop (Progress Task) and Video and Code submission (Task 4). The selected teams were sent a PlutoX drone[5] along with hula hoops. PlutoX drones are connected wirelessly to a laptop and controlled using ROS commands, same as how a drone model is controlled in V-REP in Stage 1. Teams are given a Rulebook[6] which gives detailed rules and information regarding theme implementation. It describes how to set the arena, rules to be followed and a formula to be used to evaluate their submissions. The arena is a representation of a Jungle as shown in Figure 6. The drone termed "Bird" must navigate through hula hoops of different colours which represents different types of trees based on the feedback from a camera placed at ceiling height as shown in Figure 6.

**Task 2 : Arena Printing and Hardware Testing**

In this task, teams print the given arena on a flex sheet and setup the arena as per the Rulebook. Teams were provided with sample videos and manuals to test the drone. This task aims to teach the participants pose estimation of hoops using ArUco markers. They were given a task to programmatically emulate hoops set in a given position and orientation in real world into V-REP, on completing which they had to submit a screenshot having both image output from the overhead camera and the top-view of the V-REP scene as shown in Figure 7. This task carried 20 marks & was graded based on successful emulation of hoops.

**Task 3 : Position Holding and emulation of real drone**

This task aims to teach the participants implementation of PID algorithm on a real drone. In this task teams had to implement Task 1.1 with a physical drone. They have to hold the position of the drone at a given setpoint with reference to WhyCon frame i.e. [0,0,20] as shown in Figure 8. They also had to emulate the drone in V-REP using what they learnt in Task 2. Students also had to submit an assignment answering various technical and implementation related questions. The submission criteria and evaluation method were the same as Task 1.1. This task comprised of 100 marks out of which 60 marks were based on the automatic evaluation of rosbag file, 10 marks to successful emulation in V-REP and 30 marks for the assignment**.**

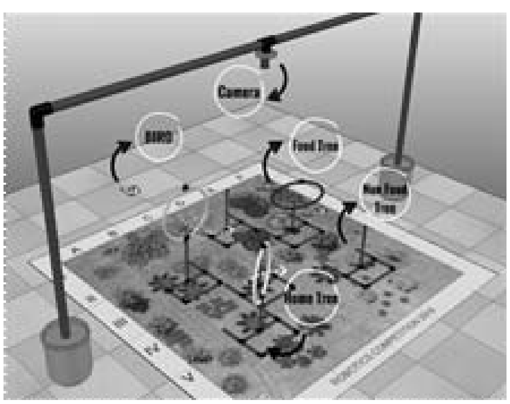


**Figure 6:** Hungry Bird arena setup

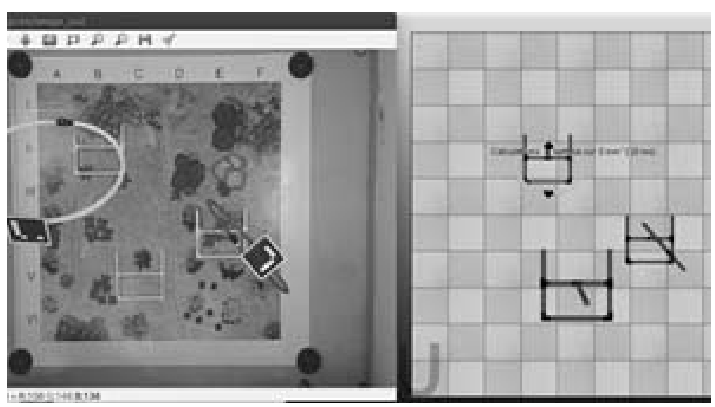

**Figure 7:** Emulation of real world in V-REP

[5] https://www.dronaaviation.com/plutox/

[6] https://drive.google.com/file/d/1IPPPzwfyCT0YsWydXEgytTdhSYa_iDfH/view?usp=sharing

**Progress Task : Traversal of drone through a hoop**

This task aims to teach the participants waypoint navigation of Nano drone through each path points in the generated global path. In this task, the teams are to control the drone to steer it through a hoop set at a given position and orientation as shown in Figure 9. The task carries 100 marks and was graded on the basis of the time taken to complete the task, emulation of the hoop using an ArUco marker, emulation of drone using a WhyCon marker, computation of the path and the number of collisions.

**Figure 8:** Position holding of Drone in Task 3

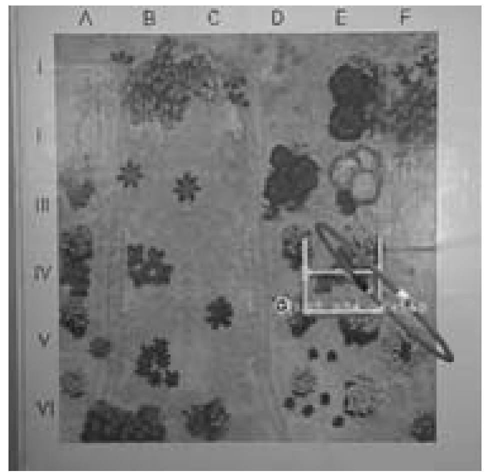

**Figure 9:** Traversal of drone through hoop

**Task 4 : Video and Code Submission**

In this final task of the competition, teams had to upload a video demonstrating their solution. Two configurations of arena setup were provided, one with less number of traversals and obstacles and a harder, optional bonus one. A run is considered successful if the drone traverses through hoops as per the given configuration without any collision. This task was graded based on a formula specified in the rulebook and evaluation of the final code. Cumulative marks of Task 4 and Progress Task were considered for selection in Finals.

**Finals : Demonstration of theme at IIT Bombay**

Six teams that demonstrated the best run (using the formula given by us in the Rulebook) for both configurations were chosen as Finalists to compete the Finals held at IIT Bombay.

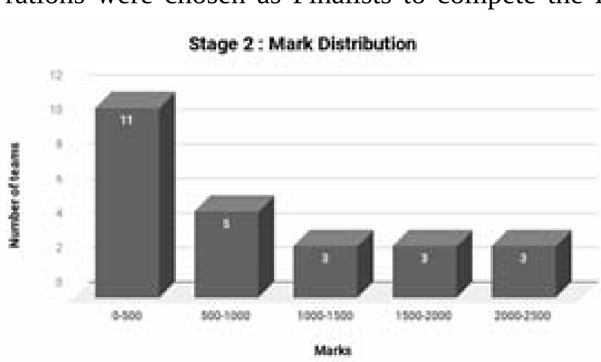


**Figure 10:** Mark distribution in Stage 2

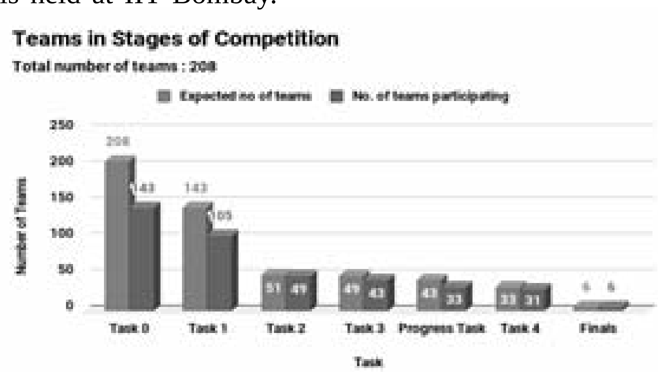


**Figure 11:** Teams in stages of the competition

## ANALYSIS OF IMPACT AND EFFECTIVENESS

The primary objective of this competition was learning control system design and navigation using Nano-drone. Through this theme, students are given exposure towards Linux, Python, Lua, ROS, V-REP, PID tuning and automatic path planning.

We started with 208 teams for Hungry Bird theme. Figure 11 presents details of the number of teams participating in various stages of the competition. Expected numbers of teams are those who have made submissions of previous tasks. For instance, In Task 1, there are 143 expected teams as they have submitted the previous task, Task 0. 51, i.e. 24.51%, of the total teams selected for the theme were shortlisted for Stage 2. Hence 51 teams are expected to submit Task 2. As six teams were to be chosen as finalists, the expected number of teams for finals are six. Number of teams participating are the teams who have made a submission for the corresponding tasks.

| Level * | Level Description * | Task | Skills Acquired |
|---|---|---|---|
| Imparting Knowledge | Recognition and understanding of facts, terms, definitions, etc. | Task 0: S/W Installation and Getting familiar with V-REP and ROS | • Installation & basic learning of V-REP and ROS<br>• Learning Pose estimation using WhyCon and ArUco markers |
| Application of Knowledge | Use of knowledge in ways that demonstrate understanding of concepts, their proper use, and limitations of their applicability | Task 1.1: Position holding of Drone<br>Task 1.2: Path planning in V-REP | • Implementation of PID to control drone in V-REP<br>• Waypoint navigation of drone in V-REP<br>• Learn to compute global path using OMPL |
| Critical Analysis | Examination and evaluation of information as required to judge its value in a solution and to make decisions/ selection of technology accordingly | Task 2: Arena printing, Hardware Testing<br>Task 3: Position holding and emulation of drone | • Implementation of PID controller on a real drone<br>• Real time emulation of drone in V-REP |
| Extension of Knowledge | Extending knowledge beyond what was received, creating new knowledge, making new inferences, transferring knowledge to usefulness in new areas of applications | Progress Task: Traversal of drone through a hoop<br>Task 4: Video and Code Submission | To have a final working demo of the drone traversing through the hoops with given configurations |

**Table 1:** Mapping level of learning outcomes to Tasks and Statistics

* Levels and Description of levels are taken from (Davis et al, 1997)

Table 1 summarizes the knowledge imparted to the participants from the competition in each task. Table 1 and Figure 11 shows that 68.75%, i.e. 143 out of 208 teams, have learned the basic concepts of ROS and V-REP. It also shows that 50.48% of total teams, i.e. 105 out of 208 teams, have participated actively in Stage 1 and have implemented PID controller to navigate the drone through the computed path in the Simulator. 24.5%, i.e. 51 out of 208 teams, had hands-on-experience with a real drone. 60.8%, i.e. 31 out of the selected 51 teams, actively participated throughout the remaining tasks and made final submission. It is interesting to note that once the student teams qualified for Stage 2, more than 75% participated throughout the tasks indicating less number of dropouts compared to Stage 1.

To learn the effect of our theme on the participants, we took feedback from them at the end of Stage 1 and Stage 2. 451 out of 832 and 141 out of 204 participants gave responses for Stage 1 and Stage 2 feedback respectively. Figure 12 shows that 77.8% of participants who have responded to the feedback feel that the tutorials were efficient enough to cover all the relevant topics involved in the theme. Figure 12 also tells that this theme has helped 86% of participants to have a better understanding of all the concepts and tools involved in the theme. Even though according to Figure 13, 71.6% of participants found the tasks difficult, more than 50% of the total teams have actively participated in Stage 1. This indicates that the tutorials and the guidance the participants received from Piazza was able to help them overcome these difficulties. 73.5% of participants (Figure 13) who feel that the response of the mentors from Piazza helped them to solve tasks further establishes that fact.

Students were also asked what were the most important things they liked about the competition. Many of them liked the way in which the problem statement was split into different tasks and the fact that completion of each task helps in bringing them a step closer towards solving the final problem statement. Apart from gaining technical knowledge, participants says that the competition helped them gain skills like team spirit, time management and leadership qualities.

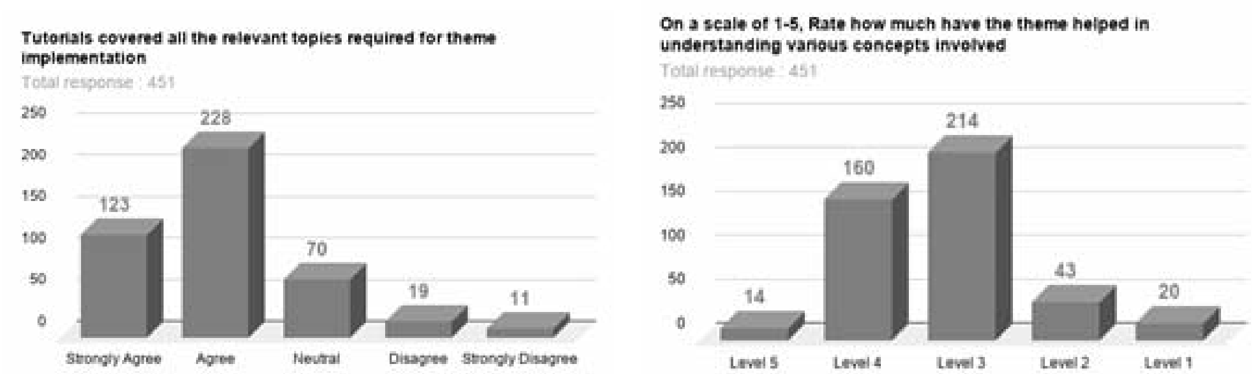


**Figure 12:** Responses from survey conducted after Stage 1

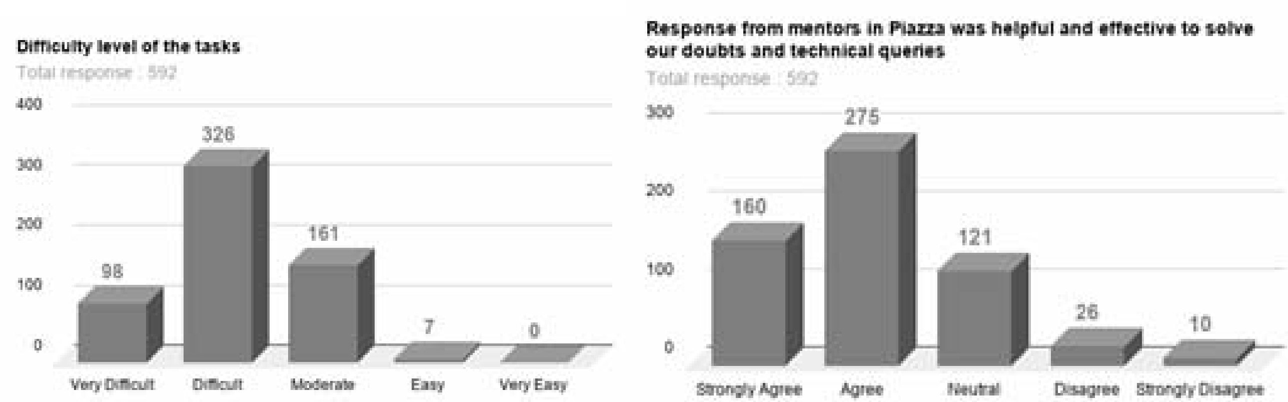


**Figure 13:** Responses from survey conducted after Stage 2

## CONCLUSION

The analysis and the feedback from the participants validates the effectiveness of our ICT-enhanced, project based learning approach. It shows that the participants through the competition had a better understanding of control algorithms and have learned tools like V-REP and ROS. Exposure towards such platforms can help them in future to validate a project in a simulator and then implement it in the real world. Also, the model of splitting the project into various tasks can help participants to address a problem or project in a similar way in future. Alongwith effectively imparting technical skills, the competition also helps in improving team spirit and leadership skills. As most of the tasks are automatically evaluated, the model also opens up the door to scale the competition to a large number of students and we are hoping to teach college students at an ever larger scale in forthcoming editions of the e-YRC. Our success is seen in the participation levels since the competition began in 2012. Registrations have grown as follows: 4384, 6324, 12428, 19568, 22608, 23728 to 28672 registrations in 2018 - in spite of a 30% YoY reduction in engineering college seats since 2015.

## ACKNOWLEDGMENTS

e-Yantra Robotics Competition is administered and managed by the e-Yantra team. We acknowledge the contributions of the following team members in devising the theme format, tasks and competition web-infrastructure: Dr Krishna Lala, Aditya Panwar, Simranjeet Singh, Ajit Harpude, Vishal Gupta, Sachin Patil, Lohit Penubaku and Rathin Biswas.

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# EXPLORING ANGLES IN A PROGRAMMING ENVIRONMENT

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*In this paper, we describe the results from a whole-class design experiment in a sixth-grade classroom where students explored angles through activity in Scratch programming. The retrospective data analysis shows that through this programming activity students were able to form advanced generalizations about angles that are not accessible with the static representations of angles on paper. These findings illustrate the power of dynamic programming environments for transforming students' reasoning about angle measurement.*

## INTRODUCTION

The understanding of angles is foundational for working with other geometric concepts such as polygons, symmetry, transformations, and for developing arguments in geometric proofs. Even though angles are essential in understanding many aspects of mathematics, students continue to struggle in understanding this concept. Previous research has found that young students develop a variety of misconceptions with regards to angles. Examples include students reasoning that an angle measure depends on the length of its sides (Smith, King, & Hoyte, 2014), or that an angle only goes counter-clockwise (Mitchelmore, 1998), or that the word 'angle' evokes a 'right angle' prototype (Devichi & Munier, 2013).

Smith et al. (2014) argued that students develop these misconceptions because angles are introduced using static representations on paper. Students are often expected to reason about the figural aspects of geometric objects when they are asked to work with the appearance of a static drawing (Hollebrands, 2003). Mitchelmore (1998) differentiated between *dynamic angles* as illustrating the motion of opening or rotation from *static angles* as the result of that motion. Research has shown that when students work with dynamic angles, for example, by modelling angles using physical body rotations (Smith et al., 2014), they can abandon their misconceptions about angles (Devichi & Munier, 2013). In addition to physical motion, students can experience dynamic angles through dynamic geometry environments (DGEs). Hardison (2018) found that when angles are presented in a DGE (e.g., Geometer's Sketchpad [GSP]) they can reason about angles as an amount of rotation. In learning geometry, it is important for students to work with figures rather with a drawing (Parzysz, 1988). When students work with digital environments, they can control or manipulate mathematical objects and identify invariant features and mathematical relationships (Hollebrands, 2003). We believe that this is the kind of reasoning that would help develop students' figural understanding of angles.

Research on technological tools shows that students adapt their understanding of a geometric object consistent with the functionalities afforded by the computer environments (Hollebrands, 2003). Studies using GSP treat angle as a property of geometric shapes (e.g., Hollebrands, 2003) or as a concept to be quantified (e.g., Hardison, 2018). For example, in GSP, a student can create a parallelogram by constructing two pairs of congruent parallel sides and verify this parallelism using angle measure. On the other hand, research on early programming environments, such as Logo programming (Papert, 1980), provided evidence that the programming environment helped students to pay attention on the direction of a turn and its measurement in degrees (e.g., Clements & Battista, 1989). For instance, students constructed a parallelogram using commands such as FORWARD motion and RIGHT turn (Clements & Battista, 1989) and reasoned about angles as the amount of turn. Additionally, the role of the Logo environment was significant for students to understand turns as conceptual objects involving iterations and directionality and construct dynamic rotations imagery (Clements, Battista, Sarama, & Swaminathan, 1996). Although exploring angles in a programming environment was found to be beneficial for students to actively construct a dynamic conception of angles, the Logo programming language was perceived as "too difficult to impact mathematics learning" (Hoyles & Noss, 1992) and discouraged further research on using Logo.

Scratch (www.scratch.mit.edu), a recent development on programming environments built based on the constructionist perspective of Logo (Papert, 1980), allows students to program interactive projects using a drag-and-drop, and snapping blocks system that encourages young students to program even without any prior programming experience (Maloney, Resnick, Rusk, Silverman, & Eastmond, 2010). Previous research on exploring students' activity in Scratch programming tasks showed that Scratch makes learning engaging and meaningful to students (Maloney, et. al., 2010) and can support students' mathematical reasoning (Benton, Hoyles, Kalas, & Noss, 2016). For instance, Calao, Moreno-León, Correa, & Robles, 2015) found that students who engaged with Scratch activities in their mathematics class have increased their understanding of mathematical concepts and processes. Considering the above, Scratch programming opens up a new opportunity for studying students' angle reasoning. Although some of these studies incorporated angles in their task design, angles are used as input values for commands on the amount of turn (e.g., Benton et al., 2016), but not focusing on students' reasoning. Consequently, our goal was to examine students' reasoning about angles as in a Scratch programming task and that this task is relevant to students' daily experiences. More specifically, we explored the following research question: How do students reason about angles as a result of their engagement with the programming task on Scratch?

## METHODS

In this paper, we present a whole class design experiment (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003) with sixth-grade students working on a Scratch task. We used a design experiment methodology to engineer particular forms of reasoning within the context of angles and investigate how these forms of reasoning developed through students' engagement with our designed task on Scratch.

## DESIGN AND CONJECTURES

The students involved in this study did not have any prior experience with Scratch. Therefore, we introduced them to some basic elements of the Scratch interface (Figure 1). First, students learned about the *Sprites*. The sprites are the programmable objects that perform the actions on the *Stage*. For instance, we used an image of an artificial satellite as one of the sprites to be programmed using the *Blocks*. These blocks, organized in the *Blocks Palette*, contain programming syntax and are shaped into puzzle-pieces that can be dragged to the Scripts Area. Also, Scratch blocks can be vertically snapped together to form a script. Part of the task is for the students to observe how the arrangements of blocks matter to the intended output. Scratch executes the codes following the order of blocks snapped vertically or wrapped by other blocks (e.g., Forever, Repeat) that repeat the commands inside the loop. We explain the difference between these two arrangements in the following paragraphs.


**Figure 1.** The Scratch interface

We designed the task "Satellite orbits the earth" (https://scratch.mit.edu/projects/196121316/) after the students have completed a module on Orbit in their science class, aiming to relate the programming task to the content they have been exploring in science. In the task, the students were asked to make the artificial satellite sprite move around another sprite, the earth, in orbit. For the satellite sprite to orbit the earth sprite, students need to create a script that moves the satellite for a short distance (e.g., 15 steps) and turns it for one degree using 360 repetitions, defining in that way a circle as a polygon with 360 sides. Our goal was to provide opportunities for students to see the purpose and utility of mathematics (Ainley, Pratt, & Hansen, 2006), specifically of angle measurement, to complete the programming task.

We asked the students to use our pre-selected blocks in Scratch to direct their attention on a particular mathematical idea embedded in the blocks (see Scripts Area in Figure 1). Similar to Logo's FORWARD or BACK (steps) and RIGHT or LEFT (turn) commands (Clements & Battista, 1989), the Scratch blocks also offer students to focus on the translation and direction of turn. Specifically, the syntax of the "move_steps" block moves the sprite a specific value for a translation (positive or negative) while the "turn_degrees" block changes the direction on which the sprite is facing. Scratch executes the value in "turn_degrees" block using the external angle. Hence, a combination of these two blocks can make the sprite move and turn. For example, the script “move 50 steps, turn 90 degrees, move 50 steps, turn 90 degrees, move 50 steps, turn 90 degrees, move 50 steps, turn 90 degrees” will create a square movement. The “Repeat _” block can be used to make the same square using a shorter script (Repeat 4[move 50, turn 90]). The “Forever” block infinitely repeats the script within the loop until the stop sign is clicked. The block “when (flag) clicked” starts the simulation, while “pen down” and “clear” tracks the path of the sprite and erases it, respectively.

**Analysis**

At the end of the experiment, we conducted a retrospective analysis to identify episodes when students focused on a particular mathematical idea or discourse (Cobb et al., 2001) in the context of angles. Then, we reanalysed these episodes to identify potentially reproducible patterns (Cobb et al., 2003) on angle reasoning as a result of students’ interaction with the task and their social sharing process in a programming activity (Papert, 1980). Specifically, our analysis was guided by themes related to turns, such as the iteration and directionality, and the role of the computer environment for developing turn concepts and dynamic rotations (Clements et al., 1996). In this paper, we present episodes of one pair of students, Paul and Laura, working on the task aiming to provide an example of the forms of reasoning that students’ exhibited.

## RESULTS

We describe some forms of reasoning about angles that students exhibited as they interacted with our task. We discuss those by giving examples from four episodes from our conversations with some students.

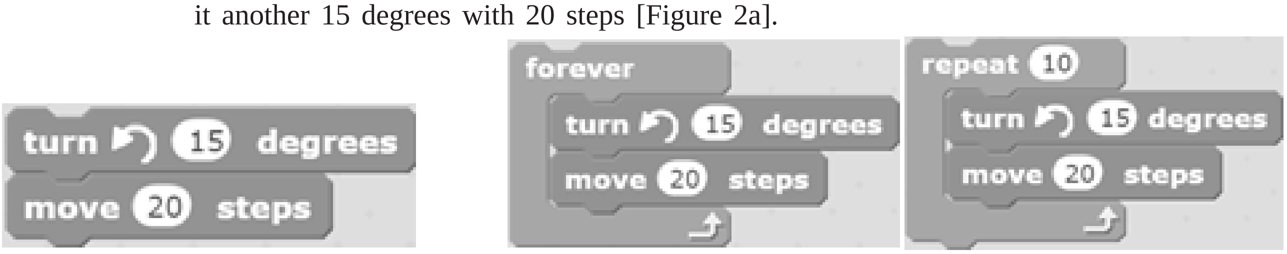
**Episode 1: Exploring angles as turns**

Students first explored the pre-selected blocks found in the Scripts Area, tried different values on the blocks and observed the change in the output. When students used only the “turn_degrees” block, they noticed that the satellite sprite was only turning in place. When they tried only the “move_steps” block, they observed that the sprite is only moving on a straight line. Therefore, they decided to combine the two blocks to observe a different output. For instance, Paul discussed his initial attempt on moving the satellite around the earth sprite:

Paul: I made it rotate!

Researcher: How did you do that?

Paul: I put it at 15 degrees that way and I put the rotation, say left to right. And then, I turned it another 15 degrees with 20 steps [Figure 2a].




(a) (b)

**Figure 2.** Paul’s script (a) before the Repeat block, (b) after the Repeat or Forever blocks

Similar to Paul, students first observed the change in the sprite when the “move_steps” and “turn_degrees” blocks are clicked individually or snapped together (Figure 2a). Connecting the “turn_degrees” and “move_steps” blocks creates a syntax using the mathematical concepts of translation and angles to move and turn the sprite. By exploring different ways to rotate the satellite and identifying the constraints of their script (snapping the two blocks together only moves the sprite once), they started building more complex scripts. For instance, Paul identified that snapping the blocks together within the Repeat or Forever block would result in the iteration of the code (Figure 2b).

Researcher: How does it work?

Paul: When I covered it [“move_steps” and “turn_degrees” blocks] with all these [Repeat and Forever blocks], it made it [sprite] repeat each one several, several times.

Researcher: How many times?

Paul: Like ten, all of these [“move_steps” and “turn_degrees” blocks]. And then it [Forever block] made it [satellite] go on and on in this one [the blocks inside the Repeat block]. So, the special, it [Forever block] makes it go forever, these ten steps. So, when I got to this, it just kept going out.

Paul built a more complex script by experimenting with the control blocks while identifying the need (Papert & Harel, 1991) to iterate the turns and create a motion of a continuous dynamic rotation (Clements et al., 1996).

**Episode 2: Expressing angle relationships**

In multiple instances during the design experiment, students were asked to describe what they have learned from working with the task and articulate the reasons behind their approach. In the following excerpt, Paul was trying to make the satellite to rotate by experimenting with the “turn_degrees” and “move_steps” tools.

Paul: It would turn that way.

Researcher: Why is it turning that way?

Paul: It’s turning this way. And it’s slowly orbiting.

Researcher: Why is it rotating like that? What do you think?

Paul: Because the degrees are too small.

Similar to Paul, students were able to generalize that the smaller the degree angle in the “turn_degrees” block (if the value in the “move_steps” block stays the same), the more time it will take for the satellite to make a full turn (Figure 3).

Another mathematical relationship that the students noticed was that the smaller the value in the “move_steps” block (if the value on the “turn_degrees” block stays the same), the smaller the track the sprite creates and the faster it will make a full turn (Figure 4).

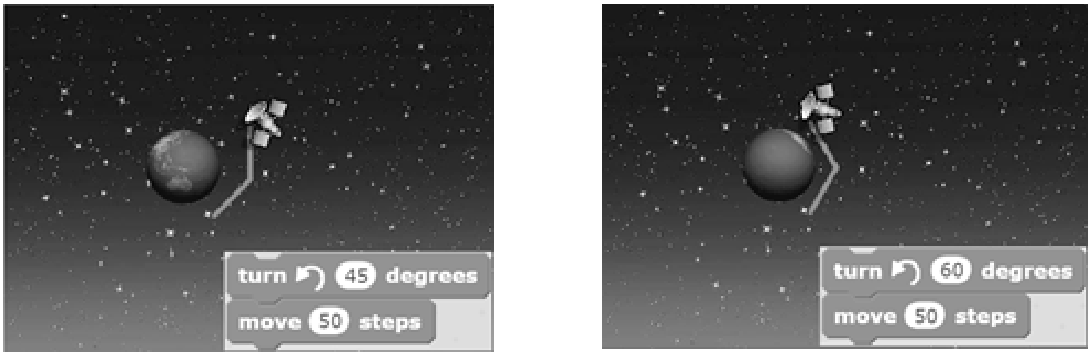
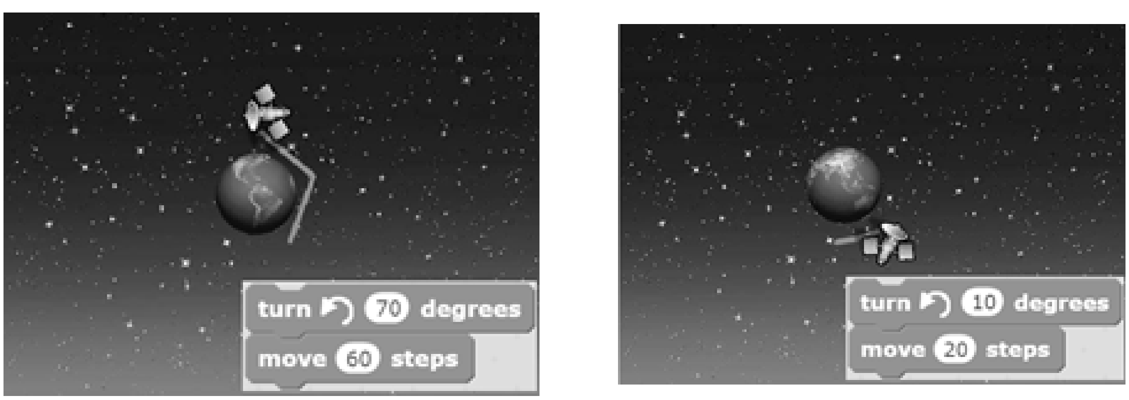



**Figure 3.** The same value for translation but different degrees of rotation


**Figure 4.** The same degrees but with different steps

As students discovered that changing the values in the "move_step" block can affect the sprite's turn, they explored different values for the "turn_degrees" block. These explorations helped students complete the task and make the satellite orbit the earth. By experimenting with different values for steps and degrees, they were able to generalize that a small number of steps and degrees will create a *smooth* circle-like movement around the earth compared to a large number of steps and degrees (Figure 5).




(a) (b)

**Figure 5. (a)** a large number of steps and degrees **(b)** a small number of steps and degrees

When students tried different degrees turn, they avoided the notion of a right angle prototype (Devichi & Munier, 2013). Also, when they tried different combinations of values for the degrees and steps, students exhibited that they do not consider angles as dependent on side length (Smith et al., 2014).

**Episode 3: Constructing codes as formulas**

Throughout the design experiment, students exchanged their ideas by collaborating and sharing what they have noticed while working on the task. We encouraged the students to interact with another student while developing their codes. The following conversation between Paul and Laura shows how students compared their work. Although both students successfully moved the satellite to orbit around the earth, their scripts were different.

Paul: Look, look. This is the formula!

Laura: I did it! [Raising right arm].

Paul: [Checking Laura's screen]. Look at mine. Mine [output] is not as fast as yours. But mine does the same job. It gets around the earth.

The excerpt above shows that Paul considered the codes he constructed as the formula to solve the task. Also, Paul compared his codes to Laura's and identified the similarities and differences between their constructed codes (Figure 6). For instance, Paul used a counter-clockwise "turn_degrees" block while Laura used a clockwise block exhibiting the bi-directionality of turns (Clements et al., 1996). By comparing their solutions, students realised that angles can turn both clockwise and counter-clockwise, avoiding the misconception that angles only go counter-clockwise (Mitchelmore, 1998). Paul also used 20 steps in the move block instead of 30 that Laura used, and as a result, his orbit was smoother and slower than Laura's as Paul expressed, "Mine is not as fast as yours." Students' independent explorations provided them opportunities to discover different ways to solve the task while closely attending to how they used the concept of angles in the programming activity.

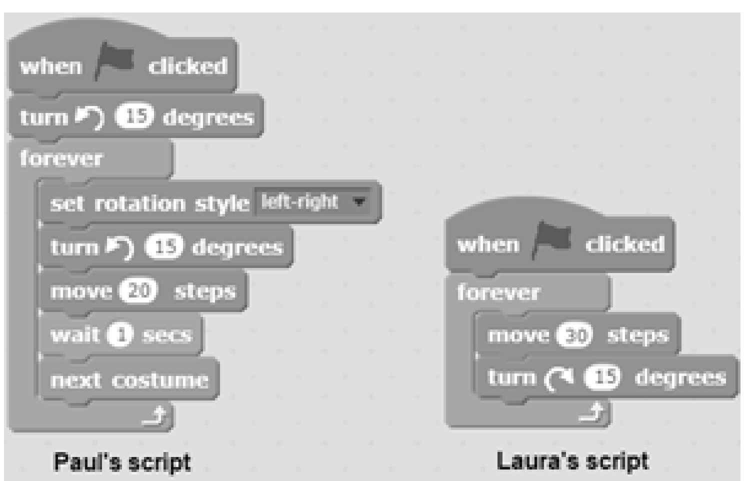



**Figure 6.** Paul's script (left) and Laura's script (right) for orbiting the satellite

We noticed that while working together to compare their work and answer questions of one another, students developed deeper understanding of mathematical ideas. They conceptualized multiple solutions to a problem by trying to understand how the solution of others work. This social sharing process is one of the benefits of undertaking constructionist activities in computational environments (Papert, 1980).

**Episode 4: Connecting ideas about angles**

By the end of the experiment, students were able to describe a full orbit as 360 degrees:

Paul: It moves back from the earth. That's the code. And I'm going to make them like I'm going to stop it, it goes back where it was. There's 360 backflip, it goes back into space from earth. That is what I call science. But I want to do it better.

In addition to his math reasoning about orbit, Paul also made an explicit link to the science context of satellite sent into space from the earth. Similar to Paul, students were able to make connections between concepts used in the programming task with the ideas in mathematics, other disciplines, or even with their everyday lives. When students actively engaged with the designed task on Scratch, they creatively integrated scientific, mathematical, and technological ideas as a learning experience meaningful to them.

## CONCLUDING REMARKS

This paper illustrated the potential of a Scratch programming task for developing students' mathematical understanding about angles. Students were able to reason about the effect of the angle measurement in the "turn_degrees" block and the number of steps in the "move_steps" block on the nature of an object's turn. They also utilized the clockwise-counterclockwise motion that an object can follow and identified that 360 degrees makes an object return to its original position. Moreover, the students learned that turns may be constructed differently but can form similar results. Therefore, we consider that integrating relevant programming activities in mathematics classrooms is important in providing opportunities for younger students to explore the dynamic nature of angles and advance their understanding of angles used in other disciplines.

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# EXPLORING STUDENTS' ALGEBRAIC REASONING ON QUADRATIC EQUATIONS: IMPLICATIONS FOR SCHOOL-BASED ASSESSMENT

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*The first encounter with abstract mathematical reasoning impedes the understanding of algebra for most students at lower secondary education levels, which continues to upper levels and beyond. A descriptive survey research involving 300 grade 11 students of ages 14 to 20 clustered into low, moderate, and high academic performance was conducted. Written responses were collected using a Mathematical Reasoning Test, regarding students' argumentation modes for justifying an algebraic conjecture and assessment of suggested solutions for a given quadratic equation. Chi-square test revealed no significant relationship between students' modes of justification and the type of school they came from, $c^2(4) = .50$, $p = .97$. However, the majority exhibited limited forms of understanding and comprehension of quadratic equations. There is a serious need for more attention to students' algebraic reasoning in school-based assessment of mathematical learning.*

**Keywords:** *Algebraic reasoning; School-based assessment; Quadratic equations.*

## INTRODUCTION

Students' success in secondary school mathematics is partly dependent on their understanding of algebra. This could be attributed to the fact that algebraic reasoning allows students to explore the structure of mathematics (Ontario Ministry of Education, 2013). In the Zambian curriculum for secondary school mathematics (Curriculum Development Centre, 2013), algebra is introduced to students at the beginning (grade 8) of their secondary education. At that level, students are expected to begin developing abstract thinking that may be required for their advancement in mathematics and science subjects. However, being their first encounter with abstract mathematical reasoning, understanding of algebra has proved to be a 'thorn in the flesh' for most students at that level. Through personal experience, Greer (2008) narrates:

It is a troubling experience to sit beside an eighth grader who is vainly trying to remember what to do with an algebraic equation and reflect that several more years of frustration lie ahead for that student (p.423).

The narration above partly suggests that such difficulties might be carried over till the end of their secondary education and later at college or university. This is also evident by reports and studies that have highlighted students' limited understanding and comprehension of algebraic concepts at both secondary school (Examinations Council of Zambia, 2018) and tertiary (Mukuka & Shumba, 2016) levels of education. Deriving and solving quadratic equations has been reported by the Chief examiner (Examinations Council of Zambia, 2016; 2018) as being challenging to most of the candidates who sat for Grade 12 national examinations. Besides that, it has been noted that students' difficulties in comprehending algebraic concepts is not unique to Zambia because similar results have been reported in other settings (see Kramarski, 2008; Lucariello, Tinec, & Ganleyd, 2014; Organisation for Economic Co-operation and Development [OECD], 2013; Susac, Bubic, Vrbanc, & Planinic, 2014).

Despite this being the case, none of the research conducted in Zambia has attempted to understand students' algebraic reasoning at secondary school level. Our belief is that understanding students' difficulties relating to algebra at grade 11 level would give teachers enough time to correct the situation before those students complete their secondary school education. In an attempt to addressing those challenges, more efforts especially in the developing world like Zambia could be channelled towards identifying classroom practices that can foster students' algebraic reasoning rather than focusing on promoting memorisation of facts. Greer (2008) and Kaput (1999) have highlighted the forms of algebraic reasoning that are relevant to schools and how school algebra can be taught. Nevertheless, our intention here is to understand students' algebraic reasoning on quadratic equations because none of the studies conducted in Zambia has made such an attempt. Consequently, results of this study will lay a foundation for further research on how the reasoning abilities could be enhanced among learners of algebra and mathematics in general.

## PURPOSE OF THE STUDY

This study seeks to understand students' ability to reason logically and to rationalise and/or justify mathematical claims. This paper reports on the results of the analysis of data that was collected from grade 11 students on a mathematical reasoning test involving quadratic equations and functions. In line with the model recently developed by Jeannotte & Kieran (2017) on the "process aspects" of mathematical reasoning, our analysis is guided by the following research questions:

i. What are the modes of argumentation used by students in justifying algebraic conjectures?
ii. How do the students assess and validate other people's solutions of a given quadratic equation?

These questions explore how students reason algebraically. Understanding how students assess and validate other people's solutions of a given equation is important to anticipate how they can generate their own solutions when they encounter similar tasks during school-based or external assessment of mathematical learning.

## METHODOLOGY

Participants of this descriptive survey research were 300 grade 11 students aged between 14 and 20

*($M = 16.24$, $SD = .98$)*. A Cluster random sampling method was used to select the participants from six public secondary schools within the Ndola district of Zambia. Schools were grouped into three clusters based on their average academic performance (high performing, moderate performing, and low performing). To ensure the representativeness of the sample, two schools were randomly selected from each of the three clusters. At each of the participating schools, one grade 11 class was randomly selected and all the students from each of the selected classes were included in the sample. Before administration of the questionnaire, permission from the relevant authority (Ministry of General Education Permanent Secretary, Provincial Education Officer, and the District Education Board Secretary) was sought and granted.

All the participants provided written consent and the study had received ethical approval from the Research and Innovations unit of the College of Education, University of Rwanda. Students' written responses were collected via a '*Mathematical Reasoning Test*' comprising of seven (7) mathematical tasks on quadratic equations and functions. However, this paper focuses on two of those tasks. Task one is concerned with students' justifications about the truth of the statement "*$x^2 + 1$ can never be zero if $x \in R$*"**.** Task two tested the students' ability to assess and select the most convincing solution of a quadratic equation $(x+2)(x+3)=14$. The development and analysis of these tasks were in line with the requirements of the Zambian curriculum for secondary school mathematics (Curriculum Development Centre, 2013) and previous studies (Brodie, 2010; Jeannotte & Kieran, 2017) on students' mathematical reasoning. All the tasks were assessed and validated by mathematics educators at different levels.

Students' written responses were analysed into categories of meaning using descriptive statistics. These categories focused on empirical or inductive reasoning versus analytical or deductive reasoning. Justification through inductive reasoning was based on citing numerical values to expressions or giving examples of numbers that can satisfy a given statement or expression. On the other hand, analytical justifications were based on logical deductions to arrive at a valid generalisation of a given algebraic statement or argument. A contingency table (cross-tabulation) and Pearson Chi-square Test were also generated to determine whether students from high performing schools answered questions differently from others. Factors that influenced the modes of argumentation by students were also identified to provide guidance on the potential areas of focus in future studies.

## RESULTS

The algebraic reasoning being implied here is linked to students' ability in making justified inferences with conjecturing, generalisation and justification being central to the reasoning process (Mata-Pereira & da Ponte, 2017). In that respect, students' algebraic reasoning was assessed based on two categories namely, inductive and deductive reasoning.

### Students' argumentation modes for justifying an algebraic conjecture

Students were provided with the following statement:

**"***$x^2 + 1$ can never be zero***"**.

Students were then asked to state whether the statement was true or false for real values of *x* and to construct valid explanations to justify their choices. Two hundred thirty-seven (237) students representing 79% agreed that the statement was true and only 33 (11%) indicated that the statement was false, while 30 (10%) of the students did not respond to the question. All the students who indicated that the statement was false attempted to substitute –1 for *x*. In their own thinking "$-1^2 + 1 = 0$" This reflects students' inadequate understanding about substituting numerical values in a given algebraic expression and their failure to square negative numbers.

A follow-up analysis of 237 submissions representing 79% of students who concurred that the statement was true revealed categories of meaning displayed in Table 1. Results show that the majority (n = 120 or 51%) justified their choice with explanations that were out of context. This was followed by those who gave explanations that were classified as inductive reasoning (n = 83 or 35%) while very few (n = 34 or 14%) argued deductively.

A further qualitative analysis of students' explanations indicated that 90 (75%) of those whose justifications were classified as "out of context" had misconceptions about real numbers. They treated real numbers as mere natural numbers. The remaining 30 (25%) of the respondents gave different explanations without any reference to real numbers. The following submissions by two of the respondents reflect such misconceptions: Respondent 1: The statement is true because real numbers are all positive like 1,2,3,4, etc. Picking any number and add 1 cannot give zero.

Respondent 2: The statement is true because of addition of 1. If it was subtraction, it can be zero because 1-1 = 0.

On the other hand, those who argued inductively justified their choices by citing specific integers or natural numbers to inform their conclusions. It was also established that all of those who argued inductively appeared to have mistaken real numbers for integers or natural numbers because none of them cited other forms of rational numbers (such as decimals or common fractions) or irrational numbers.

Among the 14% who were able to justify deductively, 28 (82%) of them explained that whatever real number may be substituted for *x* whether negative or positive, the square of such a number will always be positive. When that positive number is added to 1, the result will always be greater or equal to 1. The remaining 6 (18%) of those respondents made an assumption that $x^2 + 1 = 0$. When they tried to solve this equation, they reached a stage where they could not compute the square root of –1 and concluded that $x^2 + 1$ can never be zero for real values of *x*.

Table 1 also displays the variations in students' argumentation modes (deductive, inductive and out of context) across the three clusters of school average performance levels. Results indicate that 67 (69%) out of 97 respondents from low performing schools agreed that the statement was true. Of this number, 36 (53.7%) justified their choices with explanations that were classified as out of context while 22 (32.8%) justified inductively and only 9 (13.4%) used the deductive mode of argumentation.

| **Students' argumentation modes** | | **School average performance levels** | | | **Total** |
|---|---|---|---|---|---|
| | | *low* | *moderate* | *high* | |
| Out of context | *Count* | 36 | 47 | 37 | 120 |
| | *% within school average performance level* | 53.7% | 50.5% | 48.1% | 50.6% |
| Inductive | *Count* | 22 | 33 | 28 | 83 |
| | *% within school average performance level* | 32.8% | 35.5% | 36.4% | 35.0% |
| Deductive | *Count* | 9 | 13 | 12 | 34 |
| | *% within school average performance level* | 13.4% | 14.0% | 15.6% | 14.3% |
| Total responses | *Count* | 67 | 93 | 77 | 237 |
| **Total sample** | | **97** | **122** | **81** | **300** |

**Table 1:** Students' argumentation modes according to school average performance level

Results further indicate that 93 (76%) of the 122 respondents from moderate performing schools agreed that the statement was true. Forty-seven (or 50.5%) of those respondents gave explanations that were classified as out of context while 33 (35.5%) argued inductively and only 13 (14.0%) used a deductive mode of argumentation. Finally 77 (95.1%) of the 81 respondents from high performing schools agreed that the statement was true. Among those responses, 37 (48.1%) justifications were out of context while 28 (36.4%) used inductive reasoning and only 12 (15.6%) used deductive reasoning.

Overall these results indicate that the majority of students from high performing schools (95.1%) rightly recognised the statement as valid compared to 76% of those from moderate performing schools and 69% of the respondents from low performing schools. Based on the results displayed in Table 1, a Pearson Chi-square test was performed to find out whether there was any significant relationship between the way students justified their choices and the type of school they came from. The Pearson Chi-square test in SPSS version 20 showed no significant association between the two categorical variables, $\chi^2(4) = .50$, $p = .97$. Although many other factors might have contributed to students' inadequate understanding of quadratic equations, it suffices to say that the way teachers taught and assessed them might have led to such a quality landscape in providing valid mathematical justifications regardless of the school average performance level from which respondents were drawn.

**Assessment of the suggested solutions for a given quadratic equation**

Respondents were presented with the following solutions for a quadratic equation $(x+2)(x-3)=14$.

**Solution A**

$$(x+2)(x-3)=14$$
$$x+2=14 or x-3=14$$
$$x=14-2 or x=14+3$$
$$x=12 or x=17$$

**Solution B**

$$(x+2)(x-3)=14$$
$$x^2-3x+2x-6=14$$
$$x^2-x-6=14$$
$$x^2-x-20=0$$
$$x^2-4x+5x-20=0$$
$$x(x-4)+5(x-4)=0$$
$$(x+5)(x-4)=0$$
$$x+5=0 or x-4=0$$
$$x=-5 or x=4$$

**Solution C**

$$(x+2)(x-3)=14$$
$$x^2-3x+2x-6=14$$
$$x^2-5x-6=14$$
$$x^2-5x=20$$
$$x^2-5x-20=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-5\pm\sqrt{-5^2-4(1)(-20)}}{2(1)}$$
$$x=\frac{-5+\sqrt{-25+80}}{2}$$
$$x=\frac{-5+\sqrt{55}}{2} or \frac{-5-\sqrt{55}}{2}$$
$$x=1.21 or x=-6.21$$

**Table 2**

Respondents were then asked to assess each of the three solutions and indicate whether it was correct or wrong. They were also required to justify their choices and to provide their own solutions in an event where they found that all the three given solutions were wrong. Only 17 students representing 6% were able to dismiss all the three solutions and provide valid justifications for their solutions.

Among the 276 students who assessed solution A, 186 (64.4%) rightly identified it as a wrong solution although most of them (97%) could not justify why the solution was wrong. The few (3%) that managed to justify their choices indicated that factors on the left hand side were not supposed to be equated to 14 as that would be the case only if the right hand side was represented by zero. Those who rated solution A as being correct (n = 90 or 32.6%) had a misconception that it was okay to equate each of those factors on the left hand side to 14. They made such a wrong choice even without trying out whether those solutions fitted into the given equation. This could be another indication that students did not understand the property that $x = p$ *or* $x = q$ if and only if $(x–p)(x–q) = 0$ for real numbers $p$ and $q$.

Of the 263 respondents who managed to assess solution B, 148 (56.3%) of them rightly recognised it as a wrong solution. The common error that was identified in this solution was the "renaming" of the middle term, $–x$ (i.e. $–4x + 5x$ instead of $–5x + 4x$ or $4x – 5x$). It was further established that solution B was the most misinterpreted one because 115 (43.7%) of the respondents recognised it as a correct solution when it was actually not. This clearly shows that those students did not pay attention to arithmetical computations, neither did they try to confirm whether those solutions could satisfy the given equation or not.

Among the 258 respondents who managed to assess solution C, 191 (74%) made the right choice by recognising it as a wrong solution. This shows that a higher proportion of those students managed to recognise what went wrong in the solution. Although a lot of things went wrong in this solution, about 90% of the respondents identified only one error (i.e. $-3x + 2x = -5x$ instead of $-3x + 2x = -x$). Very few (10%) of them went ahead to look at those errors committed when substituting the values of the constants ($a$, $b$, and $c$) in the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

All of the 67 (26%) respondents that wrongly identified solution C as being correct indicated that the solution was correct because it was the only one that utilised the correct formula for solving quadratic equations. This group of respondents made such a choice without any attempt of looking at how the formula was utilised. It was also quite surprising to learn that this proportion of students at their level (grade 11) could not even see that $-3x + 2x = -5x$ was wrong. This is a confirmation that some difficulties encountered at primary and junior secondary levels regarding integer addition might have persisted even during their senior secondary education.

## DISCUSSION AND IMPLICATIONS OF THE FINDINGS

Considering the nature of mathematical tasks presented to participants, they were expected to provide logical justifications and arguments when explaining the validity of the given conjectures or claims. Although this expectation might sound quite odd since it is not stated whether students had been taught to provide such justifications, it is in line with what the curriculum demands (Curriculum Development Centre, 2013). However, responses from a majority of participants reflect an inadequate view of the nature and function of algebraic reasoning in mathematics (Ontario Ministry of Education, 2013; Van de Walle, Karp, & Bay-Williams, 2011). Students' responses to the given tasks were an indication that most of the work given in their classrooms was more of integer solutions to quadratic equations. Classification of the solutions of quadratic equations based on the value of the discriminant seemed to have been rarely discussed in those classrooms. We concur with other scholars (e.g. Brahier, 2016; Brodie, 2010; Small, 2017) that classroom-based assessment should not be limited to memorisation of facts but to enable students to make conjectures and develop formal or informal arguments declaring or supporting why they believe something is true or false.

Additionally, more than half of the students could not justify why the statement "*$x^2 + 1$ can never be zero for the real $x$*" is true. We found that most of those who failed to construct valid justifications about the truth of this statement had misconceptions about real numbers. The concept of real numbers is usually discussed in grade 8 (Curriculum Development Centre, 2013) but we found that grade 11 students expressed limited understanding of what constitutes real numbers. Substitution of numerical values into a given algebraic equation also proved to be challenging to most students. Some students refuted the algebraic conjecture because of their failure to square negative numbers. They ended up with not knowing that. This quality landscape also reflects teachers' failure to emphasise the importance of signs when manipulating algebraic expressions and number concepts.

On solution validation, refutation and assessment practices, majority (more than 56%) rightly identified the three given solutions to be (as being) wrong but only 6% of them managed to justify why those solutions were wrong and went ahead to provide their own correct solutions. One inference that can be drawn here is that learners might not have been exposed to such kind of questions. We are of the view that asking students to evaluate and validate suggested solutions of different equations is another way through which teachers could understand students' reasoning abilities and their misconceptions of quadratic equations and algebra in general.

It was also established that some of the student errors and misconceptions were not only due to teachers' 'inappropriate' instructional and assessment approaches. We observed that some students' algebraic reasoning abilities were quite low such that it would be difficult for them to solve algebraic tasks requiring higher order thinking. This is why Greer (2008) suggested that "students should be encouraged to study algebra in the spirit of keeping options open, given its status as a gatekeeper to many educational and economic opportunities" (p.427). In other words, failure to pass algebra in school mathematics should not be encouraged, neither should it be an impediment to a student's educational advancement because some careers may require a great deal of algebra while others may only need some elementary algebra.

## CONCLUSION

The main sources of student errors have been attributed to the way teachers teach and the way they assess learners. There is a discrepancy between the demands of the curriculum and the way it is implemented. Teaching to make students pass the examinations has accentuated memorisation and recall of facts among students in most Zambian secondary schools. This demonstrates the need to base teaching and assessment methods on evaluations of how students reason algebraically, and on how they communicate that reasoning to others. To reduce the backwash effect of examinations, teachers ought to discuss with students why quadratic equations are important and how knowledge of algebra or mathematics in general would enable them to solve real world problems. A great deal of research is needed to determine how teachers can ensure that learners are conversant with the functions and characteristics of algebra and how such knowledge could be applied in real life situations.

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# MIDDLE-SCHOOLERS PRIMED TO REASON COUNTERFACTUALLY ASK MORE INTERESTING QUESTIONS

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*Counterfactual reasoning is a crucial component of scientific inquiry, giving an investigator the ability to design numerous (thought) experiments given a phenomenon. It affords learners the ability to understand complex chain(s) of causal attributions by letting them think about situations where existent factual event(s) are countered in a systematic manner, either by reference to existing data, or by designing new experiments. In this work, we assessed whether asking 5th and 6th grade students (10-11 years old) to reason counterfactually results in a measurable difference in the way they pose questions about a scientific phenomenon - 'biological adaptation'. Our empirical results indicate that the intervention does make a significant difference in the nature of questions asked. Our results have implications for inquiry-based learning, emphasizing the deployment of counterfactual reasoning in science curricula.*

## INTRODUCTION

Any natural system, be it an organism, an ecosystem, or even an inorganic crystal, arrives at its particular nature through the interaction of multiple factors. Trying to identify how the system would be different if any one factor was changed, *ceteris paribus,* is an excellent way of trying to understand the influence of that factor on the overall system (Minner, Levy & Century, 2002). Thus, the teaching of science offers fertile ground for the application of counterfactual reasoning.

But what exactly is counterfactual reasoning? It is the ability to reason by considering alternatives to the existing fact, or in other words, it is thinking with a 'what if' (Roese, 1997). To illustrate, in an illuminating developmental study, Rafetseder, Cristi-Vargas and Perner (2010) told both adults and 6 year old children a story. A mother could place candy on either the top or the bottom shelf of a cabinet. If she places it on the top shelf, her tall son can reach the candy, but he can't bend down to reach it if it is on the bottom shelf, because he's recently had a fracture and his leg is in a cast. Her small daughter can only reach the candy if it is placed on the bottom shelf but not on the top one. When the researchers asked adults what would happen if the candy was placed on the top shelf and the girl came into the kitchen, they were able to answer correctly 100% of the time. However, only 24% of 6 year olds could answer such counterfactual questions correctly.

This finding exemplifies a view commonly held in developmental psychology, that the ability to reason counterfactually has a significant maturational component and does not arrive at adult levels of competence until about 12 years of age (Rafetseder, Schwitalla & Perner, 2013). However, in contrast, other scientists have discovered the ability to reason counterfactually in children as young as 4 years old (Beck, Robinson, Carroll & Apperly, 2006). It seems reasonable to conclude from the literature, therefore, that a continuum of ability to reason counterfactually exists in children between the ages of 4 and 12 years.

From the educator's point of view, what matters is when the ability to reason counterfactually can be considered *sufficiently* mature to incorporate into classroom praxis as a pedagogical instrument. Given the crucial nature of counterfactual reasoning to scientific inquiry (Kuhn, 1993), this question is of even greater interest to the science educator. In this paper, we make an attempt to introduce counterfactual reasoning into a science classroom at the middle school level and empirically evaluate the consequences.

## METHODS

### Sample

30 students (14 F, Average age = 10.5 yrs) from an elite English medium private school in Ahmedabad, Gujarat participated in the study. Two groups (A & B) of 15 students (7 from Grade 5, 8 from Grade 6 in Group A, and 8 from Grade 5, 7 from Grade 6 in Group B) were formed. Students were randomly assigned to the groups based on the order of appearance of their names in the attendance register (1st name assigned to A, 2nd to B etc.).

We used the scores of a recent class test in science to determine whether the two samples had varying levels of ability in the subject and/or intelligence differences. Even though the mean score for Group B (16.5/25) was slightly higher than for Group A (15.9/25), a two-sample T-test rejected the hypothesis that the two groups differed in ability ($t_{28}$ = -0.353, p = 0.27). Thus, at least in a statistical sense, the two samples were ability-matched.

### Protocol

The study was conducted on two different days for the two different grades (5 & 6), owing to their separate class schedules. Students from the same grade were divided into two groups (A & B) respectively. The two groups underwent a series of tasks (phases) during the study.

As illustrated in Figure 1, the study had 4 phases in all. Phases I, II & IV were common to both the groups, and are described in detail below. Students in group B underwent an additional Phase III which consisted of a counterfactual reasoning task.

In Phase I, students were shown four physical models (M1, M2, M3 & M4) of plants and animals -2 plants (cactus, water hyacinth), 2 animals (earthworm, rabbit)) to individual students. They were asked to list down the number of structural features (adaptation) that they could observe. In front of each structural feature, they were also asked to write down their understanding of the functional significance of that particular structure. In Phase II, students were provided with information cards on four different ecosystems (marine, alpine, desert and underground- E1, E2, E3 & E4 respectively). Students were asked to perform a match between

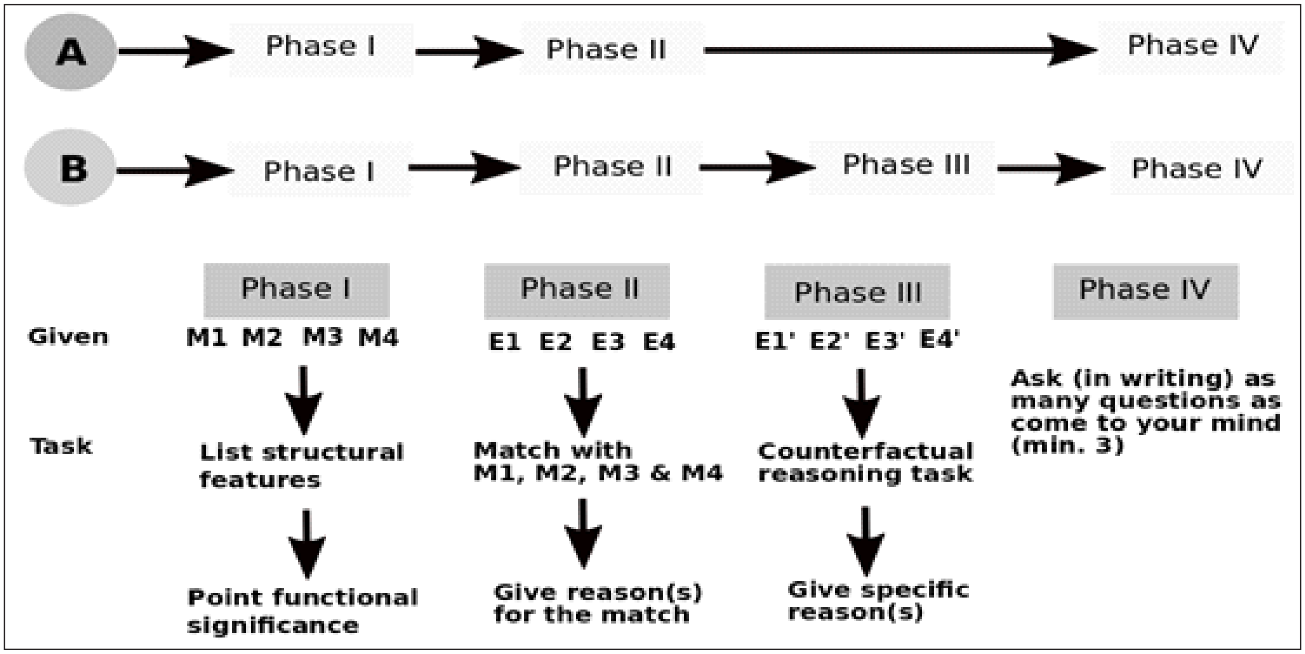


**Figure 1:** Study Design. See text for details

the model organisms from Phase I and the information cards provided in this phase (II) with minimum 4 reasons for their match.

Phase III consisted of the counterfactual reasoning task, which was undertaken only by Group B students. In this task, students were provided with pictures (but not models) of four new biological organisms alongside a description of the ecosystem they belong to (E1', E2', E3' & E4'), with the set of ecosystems used the same as in Phase II. Students were asked to think about specific structural modifications that would go for each organism that would make it suitable to survive in a different ecosystem. For example, if an organism "a" belongs to ecosystem A, organism 'b' belongs to ecosystem B and organism 'c' belongs to ecosystem C, then students may be asked to think about making structural changes in 'a' to make it survive in B and so on. Students were further asked to give reasons to support their structural modifications.

Phase IV, undertaken by Group A participants right after Phase II and by Group B participants after Phase III, involved a retrospective recapitulation of as many questions as the student could remember (minimum three) occurring to them throughout the activity.

**Data**

In this work, we present our analysis of questions posed by students in Phase IV of the study. A total of 162 questions were contributed by our 30 student sample across both groups. Interestingly, both the groups posed around same number of questions – 82 questions posed by Group A students and 81 questions by Group B. The entire set of questions were digitized, typographically (but not grammatically) corrected. Sample questions are shown in Table 1.

| Group A | Group B |
|---|---|
| Why does in cactus white things come out? | Why aquatic plant leaves are broad and wax coated? |
| How did you get this rabbit? | Why does an earthworm have a slippery body? |
| Why does rabbit have a bushy tail? | What type of eyes do fish have that allow them to see in water? |
| Why fern look like a Christmas tree? | How do spines grow in cactus? |
| Why doesn't worms have legs and arm? | How do plants in the last zone of the sea survive with less sunlight? |

**Table 1:** Examples of questions asked by both student groups

**Analysis**

We analyse our student-generated questions to see if priming with counterfactual reasoning task has had an impact on their quality. We categorize students' questions into multiple groups. This categorization is based upon accepted question-classification protocols reported in education literature (for instance, see Chin & Chia, 2004). We use multiple categorization protocols in order to make meaningful inference about the quality of questions asked to the effect of finding reasonable difference(s) between groups A & B, if any. Below, we report the protocols and the codes we use for categorization:

**a) Chin and Chia classification**: This categorization is based upon Chin & Chia (2004). We refer to this categorization protocol as the ***CC*** protocol. The categories, codes and corresponding examples from our study are given below:

(1) *Information-gathering question* ***(G)*** which pertain to mainly seeking basic factual information & whose answers are relatively straightforward, viz., 'does water hyacinth only grows in water?' (sic).

(2) *Bridging question* (***B***) that attempt to find connections between two or more concepts. For example, 'why ferns has their leaves in different rows why they can't be like normal leaves?'(sic). Here, the student is trying to link fern leaves with the concept of orientation of leaves of other plants.

(3) *Extension question* ***(E)*** which lead students to explore beyond the scope of the problem resulting in creative invention or application of prior knowledge. For example, 'why cactus store water?' (sic). Here, the student extends her prior knowledge about cactus storing water to know the reason behind it.

(4) *Reflective question* ***(R)*** that are evaluative and critical, and sometimes contribute to decision-making or change of mindsets. We use it to categorize questions which refer to some form of abstraction about a feature or function of the organism. For example, 'how water hyacinth grow in water?' (sic).

Questions that cannot be reasonably coded in any of these categories are coded ***not applicable (N)***. For example, 'why the lab(e)o fish has rhombus design on it?' (sic)

**b) Chin and Kayalvizhi classification:** This categorization is based upon Chin and Kayalvizhi (2002). We refer to this protocol as the ***CK*** protocol. The categories, codes and corresponding examples from our study are given below:

(1) *Investigable question* **(I),** where questions could potentially be answered by the student by following the scientific method. For example, 'what is inside fern?' (sic).

(2) *Non-investigable question* **(N)**, where questions could either not be answered, or were simply probes for factual information. For example, 'what is the white liquid inside cactus?' (sic).

c**) 5W1H Model:** Finally, a very general semantic categorization of questions - the 5W1H model (for 5 W's: who, what, when, where, why & 1 H for how), traceable all the way to classical antiquity in its provenance - can be applied in any information-gathering setting, including ours. We categorize questions as *why (Y), what (T), where (R), when (N), and how (H)* following the classical protocol, but add an extra category for questions requesting *statements of properties (S)*, e.g. 'is cactus poisonous'?

We also found that at times there were two parts to a question; one was a leading question and the other had either the 5Ws or H posed. In such cases, we coded only for the leading question. To build intuition for the relative strengths and weaknesses of these categorization protocols, we display some sample categorizations in Table 2.

| No. | Question | CC | CK | 5W1H |
|---|---|---|---|---|
| 1 | Why can humans live in almost all places but most animals can live only in certain habitats? | B | I | Y |
| 2 | What is the use of long horns in Arabian Oryx ? | E | I | T |
| 3 | Why ferns have so short leaves? | R | N | Y |
| 4 | Why does frog have blue blood? | E | I | Y |
| 5 | Why does the cactus have thorns? | R | I | Y |

**Table 2:** Sample questions from both groups alongside their coded categorizations

Question 1 in Table 2 is coded a *why (Y)* question in the 5W1H protocol because of its semantic intent. It is also coded as an *investigable (I)* question in the CK protocol because the underlying premise can be scientifically investigated, unlike the premise of Question 3, for example, where the premise is a subjective value judgment (N). Question 1 is also coded as a *bridging (B)* question in the CC protocol, since the student appears to be bridging the concepts of survivability and adaptability with the question, unlike say in Question 2, where the student is extending her prior knowledge about the oryx's long horns to understand its purpose (hence coded *extension (E)* question). The code for each protocol for each question was independently coded by two researchers with background in cognitive science & biology education respectively. With an initial 80% match in the coding, the discrepancies were sorted via discussion and a consensus was reached to prepare the final code.

Statistical hypothesis testing of group effects on categorization were conducted using two-sample chi-square tests to quantify the likelihood of whether the categorizations of questions resulting from both groups could stochastically have been sampled from the same underlying discrete distribution.

## RESULTS

Our primary hypothesis was that we would find differences in the nature of questions emanating from groups

A and B because of the additional counterfactual reasoning task performed by students in group B but not group A. Below we illustrate the group-level categorizations obtained via each of the three different protocols.

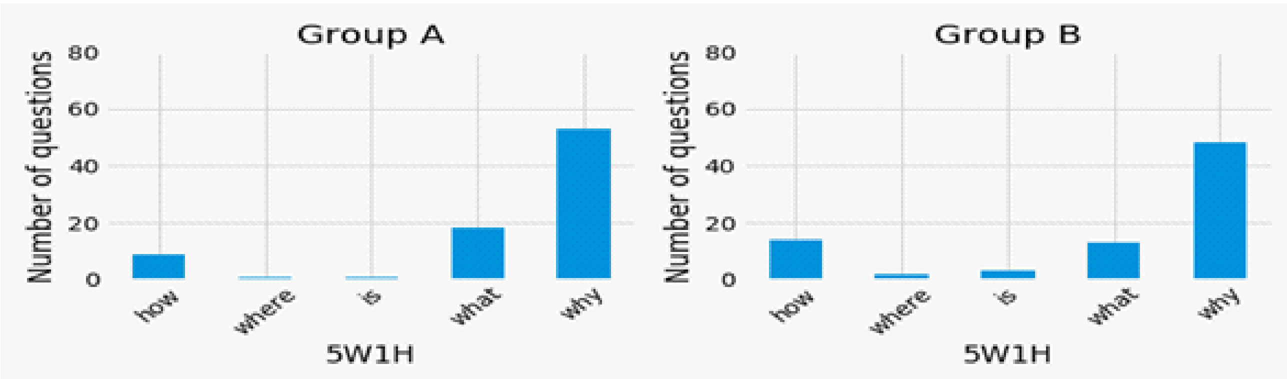


**Figure 2:** Categorization of students' questions using the 5W1H framework

It is both visually apparent (Figure 2) and supported by chi-square testing ($\chi^2 = 3.45$, $p = 0.49$) that the semantic categories of questions asked by both student groups are virtually identical. This is a reassuring observation, since it supports the case that any differences found between the two groups will not be a function of language proficiency.

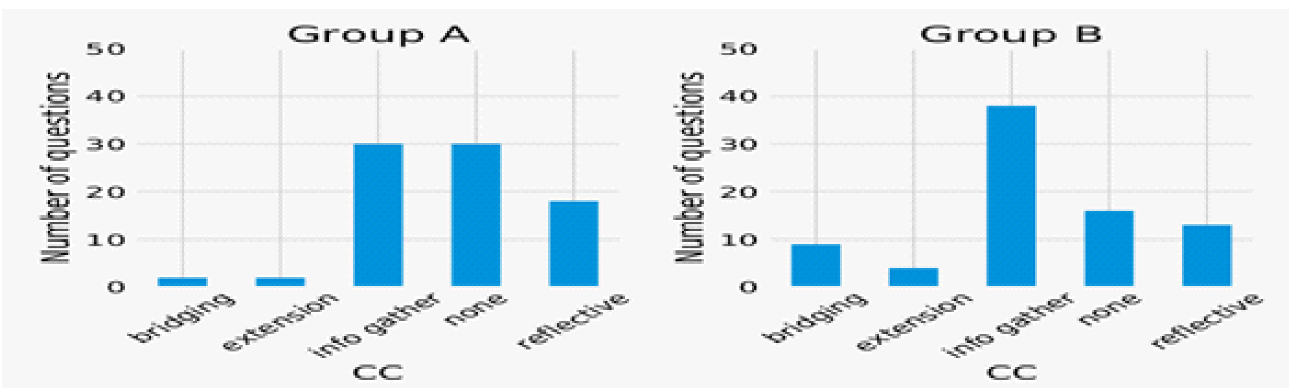


**Figure 3:** Categorization of students' questions using Chin & Chia's framework

Both visual inspection (Figure 3) and statistical testing ($\chi^2 = 11.11$, $p = 0.025$) identify significant differences in the question categories seen using Chin & Chia's categorization protocol. In particular, the use of counterfactual reasoning appears to have stimulated the generation of considerably more questions seeking to bridge between concepts (11% of all questions for Group B versus 2.5% of all questions for Group A). Growth is also seen in information-gathering questions, with a corresponding reduction in questions that could not be placed in any category in the CC framework (37% of all question for Group A versus 20% of all questions for Group B).

However, we did not find any significant difference statistically ($x^2$ = 0.88, p = 0.34) in the CK protocol (Figure 4), perhaps due to the limited number of categories present in this protocol.

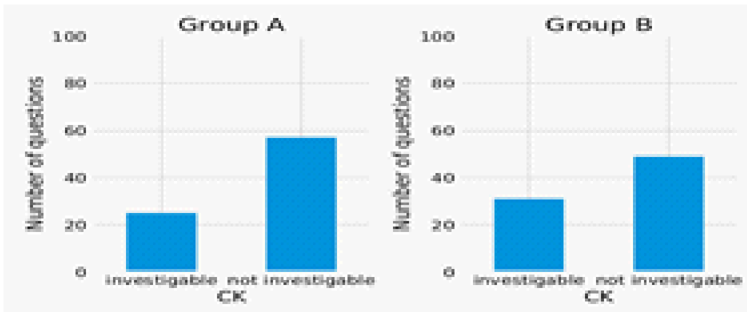


**Figure 4:** CK protocol Q categorization

The overall gist of our results is that there is a discernible change in the nature of questions being asked by Group B students. The counterfactual reasoning intervention used appears to have stimulated at least some of these students to ask questions seeking to '*bridge*' (see CC protocol) concepts presented in the activity material with other concepts.

Table 3 presents all eight (one question was repeated by two students) unique bridging questions asked by Group B students to bring out the fact that these questions are, in fact, interesting and likely to stimulate deeper understanding of associated concepts. Counterfactual reasoning (italicized) is evident in several of these questions, suggesting that the reasoning task given to the students directly contributed to the change in question pattern.

| | |
|---|---|
| How can the fish keep its eyes open underwater? | Does the sea lion's nostrils close automatically when they dive into the water? |
| Why instead of germinating on the dark forest floor, their seeds germinate high up in the mature tree whereas tiny seedlings can get light? | *Why do sea lions propel why do not they have fins just as the fish?* |
| *Generally fishes and aquatic animals have thin skin and less weight, but seal is heavy still is a good swimmer. How?* | *Why can humans live in almost all places but most animals can live only in certain habitats?* |
| Does the pitcher plant have slippery surface so that insect slip inside ? | *Why do Arabian Oryx don't live in Arctic or Boreal region just as rabbit?* |

**Table 3:** Bridging questions asked by Group B students

## DISCUSSION

In this paper, we report results from an experiment seeking to identify whether the use of counterfactual reasoning as a learning device for 10-11 year old Indian school children was likely to result in measurable pedagogical benefits. Using a controlled across-subjects design, and a suite of question categorization protocols, we demonstrated a significant effect caused by the use of a counterfactual reasoning activity on the quality of questions asked by students. We also established that the change in quality is not because of semantic changes, but because students become more likely to ask questions that seek to bridge their understanding across multiple concepts.

The empirical study reported in this paper was limited both in scale and scope. Replications of our results for large samples, over longer time-scales, and using more intensive intervention strategies (multiple sessions instead of single ones, bridging multiple concept sets instead of just one set) are clear directions for future work. However, given the paucity of studies on the efficacy of counterfactual reasoning on school students' understanding of scientific concepts, the present work may serve as a stimulant for further activity.

Our results suggest that counterfactual reasoning ability is sufficiently developed in middle school students to integrate activities built around it in science curricula, potentially within the existing ambit of inquiry-based learning (Marx et al., 2004). In doing so, they are in concord with a large array of results from cognitive psychology in the current decade pointing to the sophisticated reasoning capabilities of even very young children that strongly support a strong reconsideration of how curricula and pedagogy for pre- and middle school should be conducted (Gopnik, 2012).

While it is beyond the scope of the present paper to comment on the likelihood of success of this larger ambition, we propose that our more modest ambition of including more inquiry-based learning using counterfactual reasoning in middle school curricula could be fairly evaluated and concretely implemented based on our results and downstream replications planned as future work.

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# KNOWLEDGE REPRESENTATION – IN EYE THROUGH EYE WITH BIRDS

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*The paper draws from an understanding that every child has an insatiable intellectual curiosity and a touch of imaginative power. Understanding students' views are vital for meaningful learning. The paper reflects on outdoor observations and drawings as an instrument in developing skills, knowledge and appreciation for the natural environment. The paper also reports some interesting findings on knowledge representation through the bird module designed by the author that focuses on a target audience of eighth-grade students. Insights from the study can be directed towards implementing outdoor learning in Indian schools.*

## REVIEW OF LITERATURE

It is to be agreed upon that, knowledge represented through experience entails learning. Learning influences thinking. Thinking draws a pattern on attitudes from space which may project/reflect through behavior. Affective-behavioural relation of the learner's connect with learning through time has been an important basis for arguing the value of experience. Krishnamurti (1947) observed that every child is born with a natural curiosity. He argues that adults need to encourage children to grow and pursue a variety of subjects by reinforcing their own need to learn. Asserting the role of experience with learning, derived from being immersed in natural settings, Tagore (1917) greatly believed that nature is a child's best teacher. A child's harmonious development will only take place if the child is given unrestricted freedom.

In the Indian context, the Position Paper of the National Focus Group (2006) on '*habitat and learning*' highlights the lack of good quality documentation on some of India's greatest environmental facets and the potentials for building a resource database involving students. The Bird Module provides a framework towards learning through experience, as becoming more meaningful. The module revolves around topics like, the relation of education and experience, learning through experiencing, stimulating learning in outdoor environments, challenges in handling outdoor learning and, linking observational and experiential learning.

## OBJECTIVES OF THE BIRD MODULE

(1) Providing an authentic experiential context to learning and scope for developing a behavioural appreciation of the natural environment. (2) Examining evidences that support and engage students in developing transformative knowledge and skills, and ecological sensitivities.

## METHODOLOGY

The study followed an exploratory design sought to understand how children relate to their surroundings and reflect on their engagement with learning activities. The study aimed to gather insights and evidence of learning through the process of engagement in different activities like drawing a bird, observing birds from a distance in natural setting and questioning.

The site chosen for the study was an English medium school called St. Johns public school, a CBSE affiliated higher secondary school in Hyderabad. The study was conducted for six days during school hours. Each session was conducted by the researcher for 45 minutes depending on the availability of students. Students from the eighth-grade were the respondents for the study, which mainly involved the age group of 12 to 14 years. This age group marks the cognitive developmental maturation within the child to understand abstract concepts. At this age students are able to comprehend complex ideas and think independently. They develop abilities to identify, observe space and situations around them. Students are also able to reason and question the workings of certain realities and situations. Besides, at this age students are also able to communicate more productively, they are able to articulate, write, read and speak comfortably which is important for their expression of opinions and ideas.

Purposive sampling technique was used in this case. The selected age group was handpicked as the most suitable sample, serving the specific needs for this study (Cohen, Manion & Morrison, 2002). The sample selected represented students from both the rural and urban places, different religious backgrounds and socio-economic settings. Firstly, the outdoor sessions were conducted in the mornings, and the indoor (classroom) sessions were conducted in the afternoons. The school followed two medium of instructions i.e English and Telugu. The sample involved about 60 students, representing gender groups, which consisted of a greater number of girls (37) and fewer number of boys (23). In the outdoor setting students were randomly selected and divided into two groups.

## DEVELOPMENT OF TOOLS

Lave & Wenger (1991) state that abstract representations are often related to the power of generality which can be irrelevant unless it is made relevant with a situation at hand. The tools used in conducting the study were designed in a way that could help probe students' ideas, thoughts and views. It was important to understand the way children think and reason. For example; Questioning, drawing, observing, writing, discussing were used as instruments to probe students to think, reason and develop new ideas. In the first two sessions, pictures and videos were used to familiarise and make students aware of the current situation and increase their curiosity.

The pictures used displayed identification of the bird, various state birds in the country, identification of a male and female bird and the habitats of different birds. Each video was presented followed by a discussion on ecology and its relation to different bird species that probed students to ask questions and express their opinions and suggestions on how they relate to the natural environment. (For example, a short documentary on ovenbirds was presented to the class. The video was used to give students a detailed visual representation on how birds build their nests). The video facilitated a discussion on the different types of bird nests and the different habitats that birds belong to. Throughout the sessions, students' doubts and questions were addressed, recorded and noted. Field observations were made by the students during the outdoor sessions. Students were also informed that the data would be collected after each session.

## PROCESS OF DEVELOPMENT OF THE MODULE: ANALYSIS AND FINDINGS

The study was organised on the basis of three themes that helped in exploring students ideas and views before and after their engagement with the module. The themes are as follows: 1) Transitions noticed in nature of questions, 2) Exploring thinking through representations, 3) Evidences of conceptual progression. The themes were organised on the basis of commonality that was captured through students' responses. The findings are captured using examples of four cases, R1= Sheryl , R2= John, R3= Fatima and R4= Rahul, which have been analysed at all stages. This is done to help in mapping the shift in the data collected before and after observations.

**Transitions in the nature of questions about birds**

Questioning is an important tool in exploring student's current knowledge, and also helps in assessing student's understanding by encouraging them to think independently. The theme focuses on exploring the kind of questions and insights shared by students before and after their active engagement with the module. A set of subthemes like, myths and characteristics of the birds were arranged on the basis of the commonalities that were captured through the questions asked by students.

**Myths and beliefs elicited in childrens' questioning**

It was observed that students were able to express their views and queries freely. It was also interesting to see the difference in the kind of questions asked before the students' engagement in the outdoor setting and after. It was observed that students began to ask more specific questions after the outdoor activity. For instance, if we look at Sheryls (R1) nature of questioning before the engagement in table 1.1, she focuses more on understanding and finding answers to questions based on stories that she probably must have read or heard previously. Her question was a very interesting one as this example showed how students partly believe what they see or hear, but are also very curious in addressing the reason behind why certain things happen the way they do. The questions also highlight students curiosity to know the reasoning attached to a myth.

Notable changes could be seen through students questions after their engagement with the environment. Evidently, the questions also highlight the nature of details students attended to, when engaged in outdoor observations.

Students were more interested in understanding specific details and features of the bird, as illustrated in table 1.1. For example, R1 wanted to know how birds communicate, she was especially interested in knowing how woodpeckers communicate or how she could identify the sex of the bird or know when the bird is a male

| Themes based on commonalities | Questions asked **before engagement** | Questions asked **after engagement** |
|---|---|---|
| | **(R1)** Sheryl | |
| 1) Myths<br><br>2) Features of the bird | 1) *Is it true that if we touch a small or a big bird. the birds family will not be able to recognise it and they will kill the bird?*<br>*2) Is it true that if we kill a bird or her babies the family will take revenge?* | 1) *How can woodpeckers communicate?*<br>*2) How can birds drink water?*<br>*3) How eagles can fly without swinging their wings?*<br>*4) Why a hen cannot fly too high?*<br>*5) Why can't other birds fly without swinging their wings?* |
| | **(R2)** John | |
| 3) Food and habitat | *My grandmother says, souls of the dead enter into the crows body and the crow will have to be fed for the dead soul. is it true?* | 1) *Why is the Indian paradise flycatcher called the bird of paradise?*<br>*2) How reproduction occurs in birds?* |
| | **(R3)** Fatima | |
| 4) Gender reproduction | *Do birds also get headaches or stomach pain just like humans do?* | *What are the kinds of materials birds use to make their nests?* |
| | **(R4)** Rahul | |
| | Why birds move their tail when sitting on a branch of a tree? | How can a bird be identified or known which bird it is once it has been spotted? |

**Table 1:** Shift in the nature of questions elicited from students questioning

or female. Similarly R3, the questions she asked before the engagement showed her interest and curiosity in understanding the world around her. For example, R3 wanted to know if birds get stomach aches and headaches, thus indicating that she was able to think and frame questions based on abstract ideas even before engaging with the activity. There seemed to be an evident shift in the kind of questions R3 asked after her engagement with the natural environment. She now wanted to know about the kinds of materials birds use to make their nests. This could be attributed to her observations and her attention skills to understand the detailing of the way nests are built and the kind of materials that support them. R2's question after his engagement with the environment was quite different from before. He was more curious in knowing more specific details on birds, for example; "how reproduction occurs in birds".

**Exploring thinking through representations**

Students use drawings to express their ideas of how they represent the world around them. Interestingly, it was observed that students created evident differences in their representations of birds before and after their outdoor engagement. The meaning drawn from the representations created by students also suggested the use of visual and conceptual analogies. Forbus, Usher & Tomai (2005) describe analogy as a powerful learning mechanism that captures the breadth of cognitive processing. During the second session, students were asked to draw/ sketch a bird through their understanding, experiences, knowledge of a bird with no further instructions on what should be highlighted or how the labelling should be done.

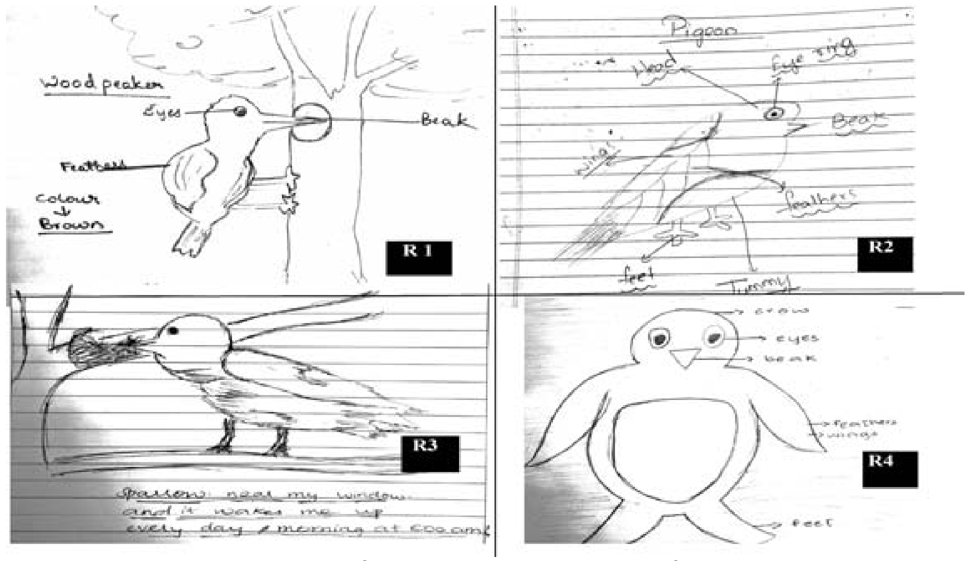


**Figure 1:** Examples of sketches made by students before engagement

Figure 1 depicts a few examples of the drawings made by students before their engagement with the environment. The sketch created by R1 suggests how the student was trying to connect the idea of bird in the context of doing something. For example; the sketch depicts an image of the woodpeckers pecking behavior. It can be noted that the students categorizing in this case is based on the activity that the bird is engaged in. This case is also a classic example of students trying to attach and depict a specific activity that a bird is usually associated with. The sketch created by R2 suggests that the child has culminated features from different species into creating the image of the bird. For example; in Figure 1 it is inferred that the student has created an image of a pigeon by using characteristics that are almost similar to fish scales or a leaf.

The second inference made here was that the student attempted to draw a visual equivalent to understand that the bird is camouflaged due to the presence of leaves around it. R2 seems to have focused on enhancing features like, the wings of the bird, the beak and its feet. R2 also highlighted the stomach of the bird which suggests the student is trying to connect the prominent features of a human body to a bird.

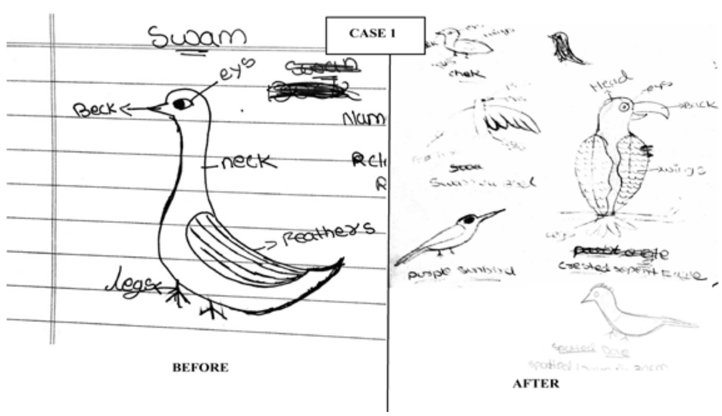


**Figure 2:** (Case 1) Drawings made before and after observations

Figure 2, case 1 displays the drawings made by Sheryl. There was an evident shift in her drawings from before and after the engagement where her observations showed up in her drawings. Some change was also observed in the way she has highlighted specific features of the bird after observations. A striking evidence was seen in the shift of the diversity of bird species included in her drawings from nature. She also seemed to have conceptualized the idea of size by portraying the relative size variations of the birds. She also described her observation by the colour and the size of the bird, the number of birds of the same species she sighted and the time at which she saw the birds. She also commented, "I did not know the name of the bird, it was jumping from one branch to another, the second bird I observed was also an unfamiliar one".

**Evidences suggesting a shift in students representation of bird species after engagement**

This study was an attempt to understand the difference in students' views about bird species (represented in drawings) before and after the outdoor observation-based activities.

| **Before observations (Day 1)** | | **After observations (Day 3)** | |
|---|---|---|---|
| Name of the bird | Number of students | Name of the bird | Number of students |
| Sparrow | 12 | Paradise Flycatcher | 14 |
| Peacock | 9 | Indian Robin | 5 |
| Pigeon | 6 | Spotted Dove | 8 |
| Parrot | 4 | Red Whiskered Bulbul | 4 |
| Crow | 4 | Drongo | 4 |
| Other birds' category enumerated: Hen: 3 Kingfisher: 3<br>Puff throated bulbul: 1, Weaver bird:1, Humming bird:2 , Ostrich:1 | | | |

**Table 2:** Evidences suggesting a shift in students representation of bird species

One of the major findings in the study was the shift in students' representations of the bird species. Table 2 shows a significant increase in the number of birds and the diversity of species observed and represented by students after their engagement with the module. During the initial stages that did not involve any observations, students represented species that were most commonly seen. These included familiar bird species such as Sparrows which were sketched by 12 students, pigeons as sketched by 6 students, Peacocks by 9 students ,Crows and Parrots by 4 students respectively. Whereas the observed species after the engagement included a more diverse array of birds which students were previously unaware about. These birds included the Paradise Flycatcher, Spotted Dove, Indian Robin, Red whiskered Bulbul and Drongo. In Table 2, "The other birds category enumerated" suggests the constancy in the names and sketches of birds represented and the number of students who sketched the birds.

Another interesting reflection was the change in the use of language by the students to describe the birds. The terminology shifted from the usage of generic terms like black birds, blue parrot and attributive adjectives like the elegant crow, colourful peacock, queen bulbul to more specific and accurate bird names for example, Spotted Dove and the Paradise Flycatcher.

## CONCLUSION AND IMPLICATIONS

The thrust of the paper is to facilitate a child's connection to knowledge, through transformative experiences –- to provide a sense of Progression. One of the major inferences made through the outdoor engagement was the transitions in the nature of questions asked by students. The finding reflects back on the principles laid down in the National Curriculum Framework, NCERT (2005) document which explains how students can import the acquired skills from their outdoor learning to classrooms. The paper also emphasizes the importance of asking questions to help enrich the curriculum.

The insights from the study helped understand students perception about birds before and after their engagement with the natural environment. The evidences emerged through the study highlight the need for exploring the conceptual and theoretical aspects of outdoor learning. The study raises numerous avenues for future investigations to deepen our understanding of incorporating outdoor learning into classrooms.

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# EXPLORING MATHEMATICAL EXPLORATIONS

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*A Mathematical Exploration is a loosely defined problem situation that has the potential to generate multiple questions, at least some of which allow for further questioning that lead to mathematically significant results, through means that are mathematical. For most students, such explorations offer a new and friendly perspective of mathematics and it is important to provide them with such educational opportunities. What does it take to enable and sustain mathematical exploration in a classroom? Based on the experience of taking four different cohorts through an exploration, this paper addresses this question and provides some preliminary answers. The larger study, of which this paper is a preliminary step, hopes to understand the exploratory process better so as to be able to come up with a 'local instruction theory' for explorations.*

## INTRODUCTION

Skovsmose (2001), in his essay Landscapes of Investigation, differentiates two different learning milieus. According to him, traditional mathematics education falls within what he calls the exercise paradigm, where a mathematics lesson is occupied with two kinds of activities–a teacher presenting some mathematical ideas and techniques and students working on some related exercises. The relative proportion of time occupied by these activities may vary, but the activities themselves don't. He contrasts this with 'Landscapes of Investigation', which invite students to formulate questions and look for explanations, rather than solve the exercises set by the teacher or the textbook and is characterised by classroom practices that support an investigatory approach. "When the students in this way take over the process of exploration and explanation, the landscape of investigation comes to constitute a new learning milieu." (Skovsmose, 2001, pp.125)

In the Indian context as well, we see this dichotomy with the policy documents envisaging the latter paradigm and the former prevailing in classrooms. The National Focus Group (NFG) Position Paper on Teaching of Mathematics talks of the need to ensure learning environments which invite participation, engage children in posing and solving meaningful problems, offering them a sense of success and to "liberate school mathematics from the tyranny of the one right answer found by applying the one algorithm taught", through a multiplicity of approaches, procedures and solutions (NFG, 2006).

Polya (1945) talks about the need to challenge the curiosity of students by setting them problems appropriate for their mathematical knowledge and helping them solve the problem with stimulating questions, thus giving them a "taste for and some means of independent thinking". Without this a student may miss out on the opportunity to know whether he/she has a taste for mathematics at all. We see mathematical explorations as a means to provide such an opportunity.

'Mathematical Exploration' is an open-ended and loosely-defined problem situation, that involves students asking their own questions, choosing the ones that interest them, following different paths to find answers and asking further questions.

The Oxford dictionary defines exploration to be "The action of travelling to or around an uncharted or unknown area for the purposes of discovery and gathering information; the action or activity of going to or around an unfamiliar place in order to learn about it; expedition for the purpose of discovery'' (OED Online, 2019). This is a relevant image for mathematical explorations by students as well. Critical to exploration is the unfamiliarity of the terrain. It might have been charted extensively by others, but the fact that it is new terrain for the student is important. Navigating the terrain for the purposes of discovery and gathering information is both challenging and engaging for the students. It is here that the availability of reliable maps, and a tour guide can make a big difference. It is also clear that the terrain may be difficult but not forbiddingly so, lest the explorer give up too early. Existence of vantage points from which one can take an overall perspective of the route travelled and pathways ahead greatly helps the exploration.

Similarly, an activity that is intended to set forth a mathematical exploration should be unfamiliar enough so as not to have a learnt solution and at the same time relevant, engaging and approachable. The entry point to the activity should be accessible to every student and ideally there should be multiple entry points. At the same time, the activity should have the potential to challenge the more interested students and keep them involved. In other words, the activity should have a 'low threshold, but high ceiling' (LTHC). The activity should have the potential to branch out into multiple trajectories, at least some of which have the potential to raise deeper questions, which the students have the necessary resources to solve. These solutions themselves could generate further questions, some of which may not be as yet answered by the general mathematical community. A Mathematical Exploration provides opportunities to raise questions, the answers to which are hitherto unknown to the explorer and sometimes even to the community at large and find answers by engaging in the processes of mathematics.

In taking over the process of exploration and explanation, students are functioning like 'little mathematicians', posing questions that interest them, and engaging with the processes of the discipline like coming up with conjectures, visualising, representing, estimating, justifying, generalising, and overall experiencing the joy of making their own discoveries. This gives them a different kind of experience of doing mathematics , different from the fearsome and anxiety-inducing one that they are used to (Ramanujam, 2010). In the learning milieu created by explorations, every child has an opportunity to succeed at some level. In the Indian context, one of the first attempts at creating such a learning milieu can be seen in Eklavya's Prashika Project, though at the primary level (Agnihotri, Khanna, & Shukla, 1994).

## RELATED RESEARCH AND ENSUING QUESTIONS

Problem posing and solving and engaging in the processes of mathematics or 'thinking mathematically' are three key aspects of a Mathematical Exploration. Understanding what is involved to enable and sustain an exploration in the classroom calls for taking a closer look at each of these three aspects and how they come together in an exploration. In addition, one also needs to understand the role of the students, teachers and the material in the process. We present a brief overview of research on these aspects.

While there has been considerable research on Problem Solving and multiple aspects of it, (Polya, 1945; Törner, Schoenfeld, & Reiss, 2007), problem posing has garnered attention in recent years as well (Brown & Walter, 2005; Singer, Ellerton, & Cai, 2015). In their seminal work on Mathematical Thinking, Mason, Burton, & Stacey, (1982) delineate the practices involved in thinking mathematically and identify specialising and generalising, conjecturing and convincing, imagining and expressing, extending and restricting, classifying and characterising as the core mathematical processes. Others (Bell, 1976; Watson, 2008; Schoenfeld, 1985) have slightly different characteristics of the processes involved.

There has also been a closer look at these disciplinary practices and characterisation of progression in mathematical thinking. Zandieh and Rasmussen (2010) and Rasmussen, Zandieh, King, and Teppo (2005) exemplify how students in undergraduate classrooms engage in disciplinary practices like defining and symbolising. These papers evolve a framework to describe the stages in their progression.

A natural question arises as to whether a similar framework can be created for secondary school students, characterising the progression of thinking and the development of disciplinary practices during explorations. Some spadework in this direction can be seen in Cai and Cifarelli, (2005) and Cifarelli and Cai, (2005). Through a case study of two college students the authors examine how an interplay of sense-making, problem-posing and problem solving sustains an exploration and initiate the development of conceptual frameworks and research tools to capture mathematical exploration processes. They identify two levels of reasoning strategies in the process– hypothesis driven and data driven. However these are not sufficient to develop a 'local instructional theory' (Gravemeijer, 2004) for mathematical explorations that describe potential learning trajectories through which students might progress, thus functioning as road maps or frames of reference for teachers who want to engage their students in an exploratory activity.

We note that the idea of a 'local instructional theory' is closest in spirit to this work. While such a theory has been developed in a specific content area, and for processes such as definition and symbolisation in the content area, our eventual goal is to develop a similar theory for explorations at the secondary school. However, while theorisation is the principal aim of this line of research, the account presented here is too preliminary for any theory-building as yet.

Jaworski's work on the role of the teacher in an Investigatory classroom, (Jaworski, 1994) identifies some generic pointers like sensitivity to students and need for mathematical challenge to sustain an investigation, but does not answer questions like - What moves on the part of the teacher help or hinder an exploration? At what stage in the progress of an exploration is an explicit hint helpful and at what stage is it a better choice to let the students struggle to find their own path? What is the nature of preparation that a teacher should have before starting on an exploration with students? These and related questions on other enabling/hindering factors for an exploration form the backdrop of this study.

## STARTING POINT AND POTENTIAL TRAJECTORIES

Combinatorics offer many rich possibilities for explorations (Maher, Powell & Uptegrove, 2011). The 'starting point' being considered for this paper is a puzzle where students are invited to arrange numbers 1-6, using each exactly once, in circles arranged along the sides of an equilateral triangle, in such a way that the sum of numbers along the three sides are equal (Trotter, 1972). That there are four such distinct arrangements provides an access point to all students into the task since they can discover these by mere enumeration and sets the ball rolling with questions such as What does it mean to say distinct solutions?, How many distinct side-sums are possible? What are the upper and lower-bounds for the side-sums?, Are these the only solutions? How does one prove that there are exactly 4 solutions? What patterns can be seen in the solutions? How can one 'transform' one solution into another? How many solutions (non-distinct) can be obtained by rearranging one given solution? How is the side sum related to the corner sum (which is the sum of the three numbers placed at the vertices of the triangle)? Can the side sum ever be twice the corner sum?

While staying with an equilateral triangle formed with 6 circles, one can ask further questions like – what if a different set of numbers are used instead of 1 – 6? Do the numbers have to be consecutive in order for solutions to exist? Will there be 4 distinct solutions, whatever the choice of numbers? If not what condition should the numbers satisfy for the existence of 4 distinct solutions? What conditions should the numbers satisfy for the existence of a solution at all?

The more interested students can engage with questions like – What if the circles are arranged in the form of a square? Or a pentagon or any other polygon for that matter (Trotter, 1974)? What if there are more than 3 circles per side? What if the circles are arranged in the form of an open curve like an S or a Z? Which of the questions asked in the context of the initial triangle are still relevant? Can the solutions/methods of solutions used there be generalised to these arrangements?

Thus we see that the 'task' meets the LTHC criteria, generates multiple questions and divergent paths for exploration. Starting from a simple puzzle also ensures the engagement of all students. It also touches upon some significant mathematical ideas like the existence of upper and lower bounds, proofs of existence or non-existence of solutions, generalisable methods that work across a range of problems etc. Students also get opportunities to engage in the processes of mathematics like observing patterns, coming up with conjectures, looking for examples or counterexamples, justifying, generalising etc. This task was tried out with multiple groups of students and similarities and differences in the way it panned out observed.

## THE STUDY GROUPS

The groups differed widely in terms of the prior exposure that they had to mathematics. Some details of the different groups are as follows:

Cohort A: about 10 students from grades 8 and 9 of a corporation school

Cohort B: about 10 students from grades 8 and 9 of a low-fee private school with students from disadvantaged socio-economic backgrounds

Cohorts C and D: about 25 students each who were part of a talent nurture camp in mathematics and from fairly affluent backgrounds

In terms of ‘mathematical background’ cohorts A and B can be considered similar, and C and D are similar as well. Cohorts A and B were drawn from ‘typical’ schools whereas cohorts C and D had been identified as potentially ‘talented’ in mathematics and had been through focused enrichment programs for a few days every year for the past 4 years. While it is interesting and important to study the impact of socio-economic context on mathematical explorations in the classroom, this paper does not take up this difficult comparative task and contents itself with the more basic question of whether explorations take place at all, across these contexts, and how they proceed. All four groups spent about 2- 3 hours on the task. This account is based on reflective notes of sessions and audio-recordings of sessions with cohorts A and B.

## THE OBSERVATIONS

The way the exploration progressed with each of these four cohorts was distinctive, even though there were pairs of cohorts with similar mathematical backgrounds. While some questions and conjectures came up uniformly across all groups, the arguments that each group came up with in support of these were different. In this section, we highlight some of these similarities and differences.

### Finding distinct solutions phase

The four distinct solutions were found out by all four cohorts, some sooner and others a little later. Different approaches to finding the solutions were seen. Though all of them started off with a trial and error method they evolved differently.

With cohort A, fairly early in the trial and error phase, after three solutions were found out, the question “Will all the numbers come as sum?” was raised by a student. The teacher, noting the potential of this question to go beyond finding solutions phase, revoiced it to the whole class. Possibly guided by this prompt, possibly not, one dominant method of looking for solutions that was seen in this group was to try to find an arrangement with a pre-determined side-sum.

Some students from cohort B engaged in a similar kind of reasoning as well, but here a student realised very early on that moving around the numbers in a given solution in a cyclic order yields another solution. Having found this, the group looked for other transformations that could be done to get more solutions from ones already found out. So for this group transforming existing solutions came to become a standard strategy to look for more solutions even when working with numbers 1-6.

Using transformations of existing solutions to generate newer solutions happened with cohort A as well, a little later, but with much excitement. The student who first thought of it claimed ‘ownership’ to the findings

by calling them ‘N’s theorem I’ and ‘N’s theorem II’ on his own, and the whole group toed the line, adopting the same terminology for the rules! They also reached the conclusion that if they find one solution another one comes free by applying the ‘theorem’ and found out some pairings happening among the solutions.
With cohorts C and D the four distinct solutions came up within the first few minutes. Though the solutions came up from different individuals, the four solutions were recorded on the blackboard very soon. The patterns among the solutions and using them to find further solutions did not emerge as points of discussion. However, these groups also had some systematic way in which they generated solutions. Cohort C for example came up with the strategy of fixing the corner numbers, and putting in the minimum of the remaining three numbers in the centre of that side where there is a maximum sum of the two numbers at the corners, and so on.

Thus, examining the different ways in which the finding four distinct solutions to the triangle puzzle panned out with four different cohorts, we notice the following:

- All 4 groups moved from looking for solutions through trial and error to better mathematical ways, though along different paths and at different rates.
- The kind of prompts given by the teacher, or the kind of student findings/conjectures chosen to highlight or revoice to the whole group may have influenced the path the exploration took.
- The sense of joy the students had in finding out something for themselves, the sense of ownership they had for these findings and how they built on these findings was evident in all four groups.

**Finding the upper and lower bounds for the side-sum**

The attempt to find ‘yet another solution’ to the puzzle soon led all the groups to the conclusion that some side-sums are possible and some are not. Soon all four groups came to the conclusion that side-sums below 9 above 12 are not possible. But they had different ways of arguing this.

For cohort A the initiation to think along these lines came from the question – “will all numbers come as sums?” The first response was that numbers 1-6 have to be ruled out. Soon numbers 7 and 8 also got added to the ‘not-possible’ lists. One of the first ‘arguments’ that came up went something like ‘3, 4 and 1 add up to 8. Now we have to have 6 somewhere and there the sum will go up. We can have a sum of 8 only on one side.” which eventually M refined to “In some circle we have to have 6. Smallest number is 1. On a side there are two more circles. To get 8 there we need to add two 1s (to the 6) and we can’t do that.” However, they did not come up with a similar argument as to why a side-sum of 13 was not possible, in spite of the teacher prompting them to follow a similar reasoning.

Cohort B also followed a similar path, trying out specific combinations of numbers and realising that they would not work and then coming up with an argument for 8 being the lower bound. Like cohort A they did not extend it to the upper bound.

Cohort C and D on the other hand did not give the specific case based arguments, but straight away gave the argument similar to that which M of cohort A had come up with to establish side-sums of 8 or less are not possible and extended it to argue that side-sums of 13 or more are not possible as well. Choosing that side of the triangle where 1 occurs, the maximum possible sum is got when the largest of the six available numbers namely 5 and 6, are on the same side, giving a sum of 12.

Later on in the course of the exploration, student N from cohort A, mentioned in an earlier paragraph, figured out a way of finding out the max sum and the min sum for any given set of numbers. He found out that the max sum was obtained when the larger three numbers occupied the corners and the min-sum when the smaller three of the six numbers were at the corners. He also saw that the smallest of the remaining three should be put in the centre of that side where the larger two numbers are at the corners and vice-versa to equalise the side-sums. Though N didn't explicitly make the connection, this results in expressions for the min sum and max sum given any set of six consecutive numbers.

Here we see that, through exploring possible and impossible side-sums, all 4 cohorts were coming to the important mathematical idea of looking for lower and upper bounds for the side-sum. But the kind of arguments that they come up with show different levels of mathematical thinking. We have students –

- trying out specific cases and concluding from them that certain sums are not possible (without a proper justification),
- coming up with a justification for the impossibility of specific side sums (say 8 and 13) and in principle, extending the same argument for smaller and greater numbers as well,
- coming up with a general expression (well, almost!) for the max and min sums in terms of the given six numbers.

**Proving that only 4 solutions exist**

This could have been done on multiple ways –

1. by exhaustively considering possibilities,
2. by using parity arguments,
3. having proved that there are only 4 possible side sums, by proving that each of these corresponds to a unique solution,
4. using algebra to establish a correspondence between possible side-sums and corner sums and use this to limit possibilities.

None of the 4 cohorts started on any of these on their own accord. Knowing that method 4 above is extendable to the variations of the problem discussed in an earlier section, the teacher explicitly cued this method, by suggesting that they 'let a,b,c,d,e,f be the numbers 1 to 6 in some order and S be the side-sum' and asking them to write an expression for the sum of numbers on each side in terms of S and a,b,c,d,e,f. With varying degrees of teacher support, cohort B, C and D came up with the relation that

$$3S=\sum_{1}^{6} n + C,$$

where C is the corner sum. All three cohorts inferred that the corner sum has to be a multiple of 3 and used this to limit the possibilities to arrive at a proof. Cohorts B and C explored the generalisability of this method to other alignments of circles as well. This exercise could not be taken to completion with cohort A for want of time.

Here we see the crucial role of the teacher in keeping the exploration going. There are times when the teacher has to make an explicit suggestion instead of waiting passively for students to come up with those critical insights. In this case, with cohort A, the teacher waited unduly long, giving indirect hints to students which they did not catch on. Consequently there was loss of time and the exploration did not progress as much as it did with the other cohorts. The sense of not being able to progress resulted in feelings of frustration in students and their enthusiasm to engage with the exploration waned. Having learnt from the frustration of students and from discussion with others, the teacher changed strategy and made some explicit suggestions in subsequent trials of the module. Thus, it is a hard task for the teacher to balance between giving just enough support so that students stay motivated and at the same time leave sufficient opportunities for them to struggle and discover things for themselves.

## CONCLUDING REMARKS

Each of the subsections of the previous section describe three ‘instances’ in the span of an exploration, each drawing attention to a different aspect that needs closer scrutiny. In the first subsection we describe the movement of all 4 cohorts from trial and error methods to more mathematical ways of solving. This raises the question: what cues (from peers, teachers or materials including tasks themselves) support or hinder this move? The description in the second subsection highlights the different levels of mathematical thinking seen, calling for a clearer description and characterisation of these levels and possible learning trajectories through these levels leading to ‘a local instructional theory’ of mathematical explorations. The third subsection highlights an instance where a teacher move or lack of it hinders an exploration calling attention to the scaffolding needed and to the timing of provided support.

Even with these limited instances, we can see some essential characteristics of explorations: realisation that there are many solutions leads to the ‘how many’ questions, and then to the ‘how does one know if there are no more’ question. Extensions and generalizations occur naturally. The difficulty is also clear: when and why does an exploration stop, or run out of steam? What learning can one carry from one exploration to another? These seem to merit a deeper investigation, and will hopefully contribute to the theorization we seek.

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# ELICITING ARGUMENTATIVE REASONING AMONG SECONDARY SCHOOL CHILDREN USING SCENARIOS AND COUNTER-EVIDENCES ON SOCIAL ISSUES

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*This study locates reasoning and argumentation within the domains of developmental psychology and curricular interventions. It then investigates patterns in arguments generated by children when faced with conflicting social scenarios. To this end, 12 students from two schools were interviewed via the Clinical Interview method. The findings that emerge from the analysis of their responses demonstrate children's tendency to argue by means of reasoning aligned with their prior knowledge and personal experience, and hold relevance for research in judgement of knowledge claims for meaning making. They also indicate that the position of authority may play an important role in elicitation of their arguments. Finally, the study outlines areas in the domain of reasoning and argumentation that exist in the scope for further research.*

## INTRODUCTION

One makes sense of their environment by means of the knowledge, skills and value sets they already possess and that are passed socially to them by means of education. Reasoning facilitates this assimilation, a process which is today understood to be indispensable in the process of defining curriculum for the generations to come (National Council of Educational Research and Training, 2006). This is where the skill of reasoning becomes significant to effect social transformation by enabling students to analyze their experiences and their learnings from those experiences. In this vein, the learner must be enabled to reason their immediate existence in the society as product and the inspiration of the disciplinary knowledge that is presented to them. Unfortunately, this enabling is yet to find representation in the way we reproduce knowledge in education. As an example of the disconnect that may exist in our education, one may consider the history of the anti-caste movement in India and the depiction of this movement within the school curriculum (Kumar, 1983; Vishwanath, 2012). One thus observes a pressing need to provide our educators and teachers with the tool of questioning and reasoning in order to effectively translate content into constructive learning.

## LITERATURE REVIEW

The discourse on reasoning amongst children today emerges out of the field of developmental psychology and builds itself against a strong reference to Piaget's work on systematic development of the cognitive processes of the child. Through his interactions with children, Piaget (1958) examined learning in children in terms of how they isolate and integrate variables in the environment.

In light of understanding environmental factors influencing reasoning, children's interaction with the 'other' in the form of peers, teachers and other adults have been studied as well. Shafto, Eaves, Navarro and Perfors' (2012) study attempts to model young children's reasoning about the knowledge and intent of the informants by analyzing their biases via a computational model of epistemic trust. Figures of authority play an important role in the practice of reasoning in children, for although they are found to not be naturally oriented towards obedience, they do understand social and institutional relevance of authority (Laupa, 1991) and their reasoning can be significantly influenced by teachers' expectation of "reasoned actions and responses" and authentic opportunities of reasoning (Diezmann, Watters & English, 2002). A deeper probe into children's reasoning through their responses to conflicting scenarios thus surfaces. With implications especially for the field of science education, these studies employ methodologies of semi-structured interviews with open ended questions and conduct conversational analyses on the "informal reasoning" employed by children in evaluation of complex socio-scientific issues via argumentation (Sadler, 2004). More studies in the contemporary research in reasoning today deem argumentation as a feasible gateway into uncovering the processes of reasoning amongst children (Kuhn, 1991). Kuhn's subsequent work (Kuhn, Cheney & Weinstock, 2000) unpacks this reasoning to reveal different levels at which personal epistemologies of children may operate (abolutist, multiplist and evaluativist) as 'knowing' of the child moves from objective dimensions to subjective ones. Fischer, et al. (2014) in their review of latest developments in the field of scientific reasoning and argumentation cite studies that depict students exhibiting "poor dialogic or social quality of argumentation as reflected in the social exchange and co-construction of arguments". However, there have been multiple techniques and models employed within classrooms that demonstrate positive results with regards to learning (Choppin, 2007; Elbers & Streefland, 2000; Forman, Larreamendy-Joerns, Stein, & Brown, 1999; Stylianou & Blanton, 2011).

In the Indian context, although the media, literature and academia are ripe with discursive conflicts as a result of a chaotic co-existence of cultures, the educational institution is one space, of many, wherein classroom spaces have been observed and reported to be didactic in nature, with little opportunities for alternative subjective expression by the students. (Educational Initiatives and Wipro, 2006; Smith, Hardman, & Tooley, 2005). Therefore, there is scope for exploration of reasoning and argumentation as it exists within the dialogic paradigm of children, with important implications for its application in the classroom scenario in the future.

## OBJECTIVE

This study aimed to:

1. Develop appropriate case scenarios (along with resource cues that constitute counter-claims) that are contemporary, familiar and present conflicting perspectives on issues of social concern.
2. Invite students to take a stance on the case through a process of reflection and reasoning by posing challenging counter-arguments.
3. Examine the patterns of argumentation emerging from the interactions with case scenarios.

## METHODOLOGY

The study employs an exploratory framework in investigation of the nature of argumentation in children. It is set in two schools in Rajendranagar, Hyderabad. The first school is a government school and the second one a budget private school. The schools were chosen as per convenience as the study did not necessitate a case-specific or theme-specific criteria for the schools. Employing convenience sampling in the study, a sample of 12 students (6 boys and 6 girls) from eighth grade from the two schools was taken. The interaction took place in English.

It was necessary that the subject of conversation stimulate intensive expression of the child's opinions, and thus, have scope for taking a stance. Three topics holding such a scope were finalized based on the criteria of: familiarity of the child with the subject, relevance of the subject and scope for development of original arguments. The first topic related to vegetarianism versus non-vegetarianism, the second concerned whether video games are good or bad for players, and the third was the increase in establishments of supermarkets and whether it is good or should local markets be preferred. For each of the topics, a repository of anticipated reasoning of participants for their opinions was created and counter arguments justified by information endorsed by various authorities were developed. These authorities were: religious leaders, religious texts, news outlets, health websites, survey results, researchers and scientists, celebrity figures, a philosopher, a journalist, authors and a policy head. These figures represent multiple knowledges and multiple ways of knowing as extant in society, and thus there was an attempt to refrain from depicting a hierarchical legitimization of any one form of knowing. This was done to enable expression of the participants' personal opinions freely. Statements and findings by these knowledge authorities were printed on to placards categorized into two sets for each topic. Each topic, then, had two sets of placards wherein one presented arguments 'for' the topic and the other 'against'. These sets then comprised the tool that was used in the one-on-one interviews conducted with the participants.

It was important for the interviews to be guided by the response of the participant as the objective of the study seeks to elicit their unrestrained opinion. The planned questions were to be kept at a minimum and open ended, and further questioning would be for the purpose of encouraging elaboration or offering a counter argument in order to allow the participant's reasoning to surface. The study, thus, attempts an application of the Clinical Interview method (Ginsberg, 1997). The administration of the interviews with the participants happened with the aid of the informational placards as described earlier. Each interview had three sections corresponding to the three topics concerned, in the order:

1. Vegetarianism vs. Non-vegetarianism (T1)
2. Video games (T2)
3. Supermarkets vs. Local Markets (T3)

Each section commenced with the participants being asked one of the following questions as per the topic:

1. (a) Are you a vegetarian or a non-vegetarian?
   (b) Do you think being a vegetarian is better or is it better to be a non-vegetarian?
2. Do you think video games are good for us or bad for us?
3. Do you think it is better to have supermarkets or local markets?

Once the participant responded with the reasoning behind their opinion, they were offered a counter-argument prepared on the placard. The pieces of information that were offered were either in direct contestation to their response, or implied the same. The interactions thus made were recorded via the sound recording application on the researcher's mobile phone and transcribed for analysis.

## ANALYSIS AND FINDINGS

An analysis of the data, identifying certain characteristics of the participants' responses is presented below. There is an attempt to shed light on their efforts to respond to a counter to their rationales, as they source that attempt from their personal beliefs (or as it may appear, epistemologies).

Table 1 depicts the number of counter-cues that were offered to the participants leading to either a complete change in the expressed stance of the participant ('Y'), no expressed shift in stance ('N') and a state of indecisiveness for a shift ('U'). Instances where the participant would express that they concede after continued countering of their reasons for their stances with information cues have been noted as Y. If their responses clearly showed that they were unyielding to the counter cues, they are noted as N. There were instances where ambiguity was observed in terms of the participants' shift in their stances. These are noted as 'U?' and depict a lack of any more reasons from the participant to justify their stance.

We can observe an increase in the average number of counter cues that the participants received over the course of progression of the interview from T1 to T3. T1 has an average of 1.91 counter cues, T2 an average of 2.5 and T3 has an average of 2.58. This may be representative of an increase in engagement on the participants' part to sustain their arguments as I presented them with more counter cues.

| S.No | Participant | M/F | No. of Counters Cues | T1 | No. of Counters Cues | T2 | No. of Counters Cues |
|---|---|---|---|---|---|---|---|
| R1 | Lavanya | F | 1 | U | 3 | U | 3 |
| R2 | Sravani | F | 1 | U? | 3 | U | 2 |
| R3 | Gayathri | F | 3 | Y | 3 | Y | 3 |
| R4 | Vishnu | M | 2 | Y | 1 | U | 2 |
| R5 | Yougesh | M | 3 | Y | 4 | U? | 1 |
| R6 | Eshwar | M | 2 | U | 2 | Y | 4 |
| R7 | Akshita | F | 2 | Y | 4 | U | 5 |
| R8 | Sailaja | F | 2 | U | 2 | Y | 3 |
| R9 | Soujanya | F | 2 | U | 4 | Y | 4 |
| R10 | Mathew | M | 2 | Y | 1 | Y | 1 |
| R11 | Mani | M | 2 | U | 2 | Y | 1 |
| R12 | Suresh | M | 1 | Y | 1 | Y | 2 |

**Table 1.** Number of Counter Cues and Shift in Stance
Y=Yes, N= No, U= Undecided, U? = Undecided but questionable

A study of the nature of their shifts within the details of their interactions shows that the shifts from one argument to another may be looked at as three different kinds. The first kind of shifts include the participants (R12 in T1 and R5 in T3) that were reticent during the course of the topic in discussion. This reflects changing of stance almost as soon as a counter cue was given. The participants demonstrating a second kind of shift (R4, R6, R9 and R11) are significant because they justified their shifts in stance by directly stating the counter cues as justification, but did not yield as readily as in the first case. The third kind of shift was from participants (R1 and R5) that gave reasons that they did not draw from the informational counter cues that I gave them, but rather came up during the process of invocation of their reasoning.

> (Lavanya, R1, has been stating that she thinks local markets are better and has been giving her reasons for her opinion. Following is the excerpt from when she is given the third counter cue.)
> Interviewer: 67% of the people are saying it's good for India, and 36% are saying that it's not. More people are saying it's good for India. Do you think local markets are better still?
> Lavanya: No, supermarkets are better. (laughs)
> Interviewer: (laughs) Okay.
> Lavanya: Supermarket have very fresh vegetables. In local market, they put vegetables on roadside. All vehicles are coming or going. That's why.

There were only two respondents (R4 and R9) who did not change their stance during the course of argumentation with them. Soujanya (R9) seemed especially persistent with her opinion (for T3) that local markets are better. I presented four counter cues to her and yet her opinion was the same. She appeared to be resorting to her personal experiences the most to strengthen her arguments. Similarly, R4 persisted with his stance for T3 as seen below:

> Interviewer: ...that lets customers scan products and pay through the app. So customers will go use their mobile phone on the...pick one from the shelf, use it, use their app, and quickly put it in their cart, and their bill is made immediately. So it's very time efficient.
> Student: In supermarket?
> Interviewer: Hm
> Student: Haan, yes sister. In local mark-
> Interviewer: Do you still think local markets are better?
> Student: Yeah, local markets only better. In supermarket, they buying... they not give extra. They wasting their times. I want this money, they requesting. Please give give...they requesting, that's not good. That's why they buying in Saturday markets, local markets.

The analysis of the participants so far, in terms of their shifts in stance, or lack of it, depicts that they do hold beliefs and opinions of their own and can make arguments to defend their stances. To this end, they may borrow from the information that is provided to them or they may utilize their own personal experience or knowledge to uphold these stances. This quality of their responses itself is indicative of the conceptual involvement that the interaction could garner from them. Their production of independent reasoning depicts an understanding and readiness on their part with respect to engagement, as shown in the case based analysis below.

| Participant Reasoning | Key words/phrases used | Vegetarian food is better | Non-vegetarian food is better | Other |
|---|---|---|---|---|
| **Personal preference** | "I like it." | R1, R4 | R2 | |
| **Health** | "More nutrition." "Proteins, vitamins." "Health" "Energy" | R1, R2, R4, R6, R8 | R3, R5, R7, R12 | |
| **Both** | "We should eat both." | | | R2, R3, R4, R5, R7, R9, R11 |
| **Villagers eat vegetarian food while city-dwellers eat more non-vegetarian food.** | | | | R4 |

**Table 2.** T1: Is it better to be a vegetarian or a non-vegetarian?
R = Respondent

For T1, five students claimed that vegetarian food is better due to health reasons, and four students claimed non-vegetarian food is better for similar reasons. Their reasons across both stances involved key words like 'energy', 'proteins and vitamins', 'nutrition', etc. (See Table 2). This sort of a clustering may be observed in their responses to the other topics as well. In the case of video games (T2), the reasons that eight participants gave for the argument that video games are bad were related to studies or completion of home-work or other work. The next type of arguments was related either to violence or health, both of which were also types that the informational cues readily countered. (See Table 3)

| Participant Reasoning | Key words/phrases | Video games are good | Video games are bad | Other |
|---|---|---|---|---|
| **Hampers studies/Homework/ Work** | "They will not study." "Cannot concentrate in studies." "Not reading" | | R1, R2, R3, R4, R6, R7, R9, R12 | |
| **Wastes Time** | | | R2, R4 | |
| **Violence** | "Interest in fighting" "Saying bad words" "Fight" | | R1, R2, R5, R9 | |
| **Health** | "Affect eyes" "Mind not peaceful" "Addicted" | | R3, R5, R9, R11 | |
| **Entertainment** | "Time pass" "Entertainment" | R7, R8, R10 | | |
| **Reprimand** | "Parents will scold us." | | R7, R9 | |
| **Can be good or bad** | | | | R9 |
| **Lack of knowledge** | "I don't know" "I don't play" | | | R1, R3, R5 |

**Table 3.** T2: Are video games good for us or bad for us?
R= Respondent

| Participant Reasoning | Key words/phrases | Supermarkets are better. | Local markets are better. | Other |
|---|---|---|---|---|
| **Distance** | "Supermarkets are far." | | R1, R5, R7 | |
| **Affordability** | "People don't have money" "Costly" "Cheaper" | | R1, R2, R4, R9, R11 | |
| **Variety** | "Variety" "All" | R3, R6 | R1 | |
| **Fresh Vegetables** | "Fresh" "Dirty vegetables" *in local markets due to* "pollution, dust on the road" "Vegetables not covered" | R1, R5, R7, R8, R10 | R12 | |
| **Economic divide** | "For rich people, they will go to supermarkets" | | | R2, R3, R4, R8 |
| **Efficiency** | "Take much time in local markets" | R3, R6, R7, R8 | | |
| **Flexibility** | | | R4 | |
| **Development** | "Farmers are poor" | | R9, R11 | |

**Table 4.** T3: Is it better to have supermarkets or local markets in the city?
R= Respondent

For the third topic, the clustering can be seen as divided between four reason-types (See Table 4). Five participants state affordability as the reason local markets are better. Five of them who, at some point, wished to argue for supermarkets being better, state the reason as availability of fresh vegetables in supermarkets as opposed to local markets where vegetables may be “dirty” due to ongoing pollution on the road. Four participants state supermarkets and local markets as being meant for people with different economic backgrounds, and thus provide no clear orientation towards either. This last group could be according a multiplist nature (Kuhn, Cheney & Weinstock, 2000) to the category “better”, suggesting that comparing of the two entities would mean different things to people from different backgrounds, but the area requires more detailed study.

## CONCLUSION

The analysis of interactions with the participants demonstrates a conceptual engagement and evidence of presence of elements of argumentation in the defenses of their stances. The argument put forth by participants in the study were also reflective, to the extent that the design of the study allowed, of the sources that the participants’ arguments originated from. The responses of the participants stemmed from their personal experiences and prior knowledge, and this observation stands in line with studies that assign domain specificity to nature of argumentation in children (Kruglanski & Gigerenzer, 2011). This insight makes it imperative for the elements of the context of the child to be explored in line with the nature of argumentation. Their arguments also brought out a certain deeper epistemological understanding of multiplist kind of ‘knowing’ in them. This finding lends a hand for teaching practices that may be based on understanding how children’s epistemologies can have a non-absolutist (possibly empathetic) nature, rendering complexity to the scope of their learning. Further research, thus, would require looking into the nature of arguments children offer in argumentation and the sources of it, sources of inhibitions that children face in scenarios of argumentation, and contextual and content-related cues and determinants of the epistemological process of meaning making in children.

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# MEANINGFUL PROBLEM SOLVING WITH SCHEMA BASED INSTRUCTION

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*Problems are crucial in physics learning because they are essential for enhancing the depth of conceptual understanding in the learner. The best means to understand a concept in Physics is by presenting that concept as the part of a problematic situation. This study examined the effects of a Schema Based Instruction on Problem Solving Ability in Physics among grade 11 students. A quasi-experimental pre-test, post-test non-equivalent group design was used. Two intact classes with a total of 114 students (60 in experimental group and 54 in control group) doing science course from two Government Higher Secondary Schools of rural background following NCERT syllabus were randomly assigned to either experimental or control group. The experimental group was taught problem solving through Schema Based Instruction, while the control group was taught through Direct Translation Strategy of teaching problem solving. Results indicated that Schema Based Instruction significantly increases the problem Solving Ability in Physics of grade 11 students better than the Direct Translational strategy of teaching problem solving.*

## INTRODUCTION

Physics students have this notion “I understand the concepts, but I just can’t solve the problems.” Young, Freedman, Ford and Sears (2004) say that in physics, “truly understanding a concept means being able to apply it to a variety of problems.” For effective and meaningful learning, the students must encounter various contextual problems. So it is meaningless to teach or learn physics concepts merely as a textual meaning or definition without giving due weightage to the problem solving process.

Researchers and educators regard problem solving as a necessary 21st century skill. In most disciplines the knowledge that is not used for problem solving tasks is too quickly forgotten within short time (Jonassen, 2010). Therefore the real goal of education in every educational context should be to engage and support meaningful problem solving. Unfortunately the traditional methods like Direct Translation Strategy for problem solving do not support meaningful problem solving. Some students and inexperienced teachers indeed think that the best way to solve problems in physics is to get equipped with a battery of equations and formulae that suits every problem situation. This notion regards problem solving as an answer getting process, not meaning making (Wilson, Fernandez and Hadaway, 2001). Problem solving is not a simple cognitive process like memorizing equations and mathematical operations. It includes a complex set of cognitive, behavioral and attitudinal components (Bautista, 2013).

Physics problems require careful analysis and interpretation of the problem scenario. The cognitive activities i.e. understanding a problem and organizing all relevant information meaningfully play a vital role in problem solving process. Jonassen (2010) argued that problem solving as a process has two critical attributes: first the mental representation of the problem (known as problem schema) and second the manipulation and testing of the mental representation in order to generate a solution. Successful problem solving requires domain of specific knowledge that includes both conceptual and procedural knowledge. Organization of conceptual knowledge and procedural knowledge of a type of problems in a meaningful pattern provides mental representation (schema) of that problem type. This mental representation of problems is the basis of successful problem solving (Chi, Feltovich & Glaser, 1981; Fuch & Fuch, 2005; Jonassen, 2004). Thus for meaningful problem solving to occur, students have to construct problem schema of the problem and to apply the correct problem solution plans based on those schema (Jonassen, 2010).

Review of earlier works on Schema Based Instruction discloses that “Schema Based Instructional Strategy” is an effective instructional strategy for promoting problem solving skill. Using a pretest–intervention-posttest–retension design, Jitendra, Star, Dupuis and Rodriguez (2013), studied the effect of Schema Based Instruction on Mathematical problem solving performance of seventh grade students. The study results demonstrate that Schema Based Instruction was more effective than students’ regular mathematics problem solving instruction. Fang (2012) considered Schema Based Instruction as one of the most supported strategy for teaching word problem solving. Jitendra and Star (2011) claim schema-based instruction (SBI) as “an alternative to traditional instruction for improving the mathematical problem solving performance of students with learning disabilities (LD)”. Schema strategy is a practical approach for training students with learning disabilities in solving word problems (Jitendra, DiPipi, and Perron-Jones, 2002)

## PROBLEM SCHEMA

The concept problem schema means knowledge structure used to identify type of problem being solved (Rumelhart & Ortony, 1977). It is the mental representation of the pattern of information that is represented in the problem (Riley & Greeno, 1988). Researchers (Jonassen, 2010; Marshall, 1995;) have studied the role of problem schema on meaningful problem solving and found that a robust problem schema includes situational characteristics and structural information about the problem. According to them, most successful problem solvers are those who can integrate the situational and structural characteristics of the problems. Problem schema could act as a facilitator for improving problem solving skills in learners. If novices learn to organize all relevant information about different problem types in a meaningful and sequential pattern, information will be effectively and easily retrieved while solving problems. So the development of an Instructional strategy that enables novices to develop the same problem schema as conceived by the expert problem solver will enrich meaningful problem solving.

In the present paper the researcher designed Schema Based Instructional Strategy for teaching problem solving in Physics and examined the effect of the strategy, to increase Problem Solving Ability in Physics among grade 11 students. The investigator also designed schema diagram of various problem types included in the selected topics (example is given in Appendix 1). The design of Schema Based instructional strategy is an attempt to integrate the situational and semantic information of the problem. The design of the instructional strategy was based on the schema theory. According to Schema theory of problem solving, the problem solving ability depends on construction and development of schema of problems.

## SCHEMA BASED INSTRUCTION

Meaningful Problem solving in physics requires not only calculation accuracy but also the comprehension of textual information, the capacity to visualize the data, the capacity to recognize the semantic structure of the problem, the capacity to sequence their solution activities correctly and capacity and willingness to evaluate the procedure that they used to solve the problem (Lucangeli, Tressoldi and Cendron, 1998). This implies that other than providing for calculations, the design of problem solving learning environment for Physics problems should include means to view the problem holistically to extract meaning from text, to construct situational model of the problem, to casually relate the data sets with structural configuration of the problem, to map the structure with readymade algorithms and to reflect upon the result of applying the algorithm on the basic premise of the problem.

In an attempt to make problem solving more meaningful, the investigator designed a Schema Based Instructional strategy. Schema based instruction is a method of teaching problem solving that emphasizes on both the semantic and mathematical structure of the problem. It utilizes recognition of key words (does a simple key-word strategy) but goes further than simple recognition to stress understanding of the situation represented in the problem (Marshall, 1995)

The composition of this instructional strategy has five basic components: the problem type, the structure map, problem schema, worked examples and practice problems. In this study the investigator classified problems in the selected topics: ‘Work, Energy and power’ in to following eight problem types: work, kinetic energy, potential energy, work-energy theorem, conservation of mechanical energy, conservation of linear momentum, mechanical power, and kinetic energy & linear momentum conservation. The classification was done based on the structural relationships embedded in the problems. The structure map is a network of the interrelationships between the different physical quantities in the problems (Gentner, 1983). Before attempting problems the learners should get acquainted with the structure map of each problem type (Example of structure map of problem type ‘Work’ is given in Figure 1)

Constructing and developing schema of each problem type is one of the crucial processes in problem solving. In this study, the investigator designed schema diagram of various problem types included in the selected topics. This schema diagram consists of the following attributes of robust problem Schema suggested by Jonassen (2010): Situation Model (consists of key features of problem scenario and its interpretation); Structural Model (represents inter-relationship between problem elements); and Arithmetic Model (represents required mathematical formula). A problem Schema represents a type of problems that can be tackled using it. Example of general framework of a Problem Schema of ‘Work’ type problems is given in Figure 2.

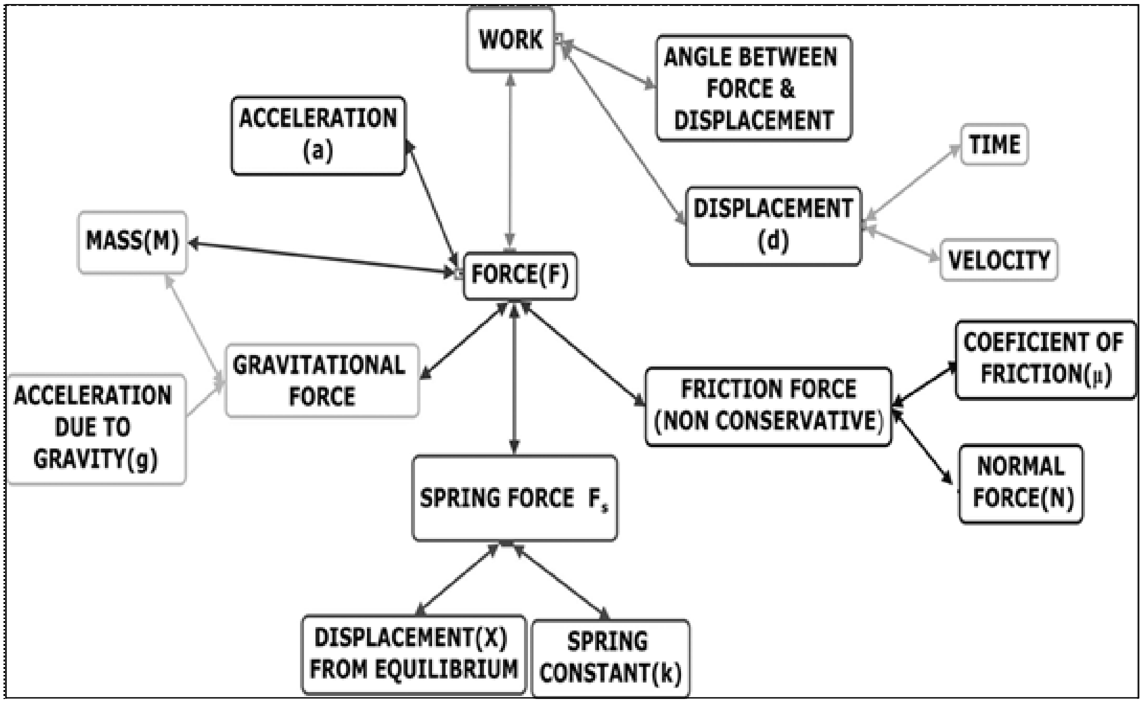


**Figure 1:** Structure map of problem type 'Work'

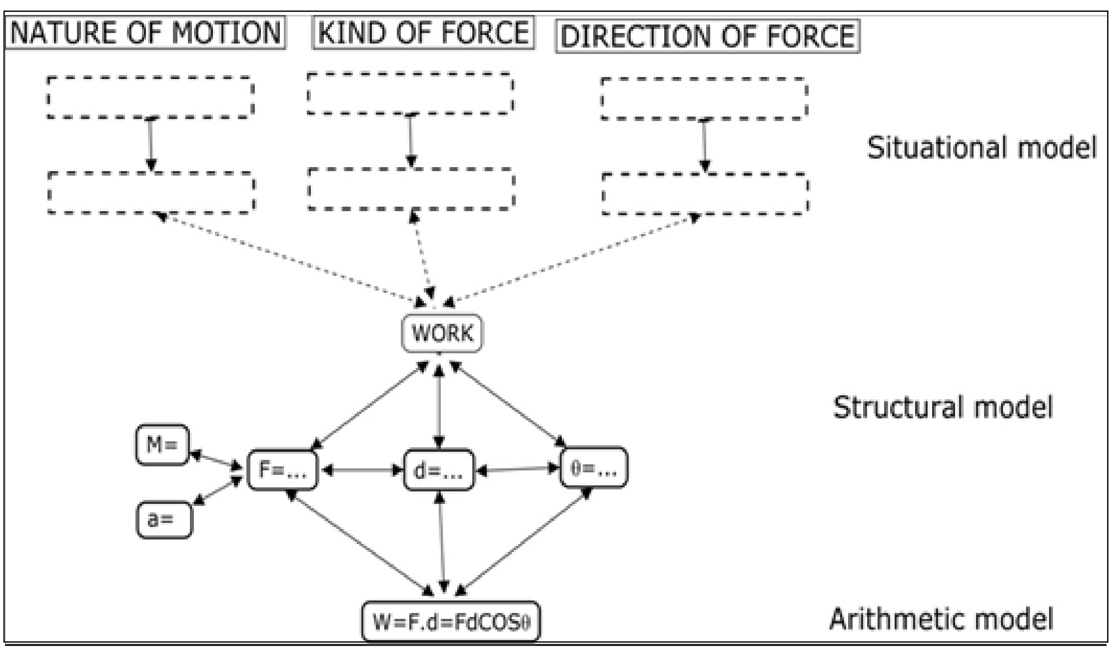


**Figure 2:** Problem schema of Problem type 'Work'

Physics problems require close reading of the statement to analyse the situations presented in it. This helps to get a feel of the problem and helps to identify the type of problem. Once we identify the problem type, the associated problem with the inclusive concepts and relations come in to our mind. This problem schema can be in the form of a readymade template for a problem type. Whereas this template serves as a basic structure to solve all problems of the type, it has to be outfitted with situational aspects as well as data sets to truly represent the problem at hand.

The 3rd component of the Schema Based Instruction is worked example. This involves hand holding the learner for walk through over the customized sequence of the schema based problem solving procedure. And the last component of Schema Based Instruction is the Practice problem. Practice problem helps learner to apply the newly learned skill in to practice. It gives them confidence to transfer the problem solving skill to new unfamiliar problems in the content domain.

The design of the Schema Based Instruction consists of the following phases:

1. Preparation for problem solving: The intention of this phase is to prepare the learner to solve problems of each problem type with the support of problem schema. In this phase Students are asked to list out familiar and unfamiliar concepts embedded in the problem scenario. They are also asked to draw situation diagram of the problem.
2. Familiarizing with problem types: The development of the situational and semantic information of a problem type and mapping all problem relevant information on to the problem schema of problem type are part of this phase. This includes understanding the concept embedded in the problem, identifying situational elements needed to solve the problem, identifying correct structural relationships in the problem, identifying the appropriate Problem schema.
3. Familiarizing with situationally dissimilar and structurally similar problems: Situationally dissimilar problems aid in recognizing the conceptual elements and to integrate it with the problem schema of the problem type. This phase includes comparing situational features with the help of situation diagrams.
4. Familiarizing with situationally similar and structurally dissimilar problems: in this phase teacher presents situationally similar and structurally dissimilar problems. Students are requested to select problems that can be solved with schema-1 (of type-1 problem). Students are also asked to formulate arguments for their selection of problems.
5. Practicing problem solving using problem schema: in this phase students are asked to practice problem solving by identifying key features of problem situation, drawing thumb nail sketch of problem scenario, identifying relevant data, identifying major concepts embedded in the problem, and identifying correct mathematical equations and operations.

## DIRECT TRANSLATION STRATEGY OF PROBLEM SOLVING

According to Jonassen (2010), Direct Translation Strategy is a “form of problem solving that typically involves reading a well-structured story problem, attempting to identify the correct equation, inserting values from the problem statement into formula and solving for the unknown value." (p.308). In this strategy students learn to "directly translate the key propositions in the problem statement into a set of computations" (Jonassen, 2010, p.28).

## OBJECTIVE OF THE STUDY

To find out the effect of instructional strategy (Schema Based Instruction/ Direct Translation Strategy of problem solving) with Non-Verbal Intelligence and Logical Mathematical Intelligence as covariates on Problem Solving Ability in Physics among grade 11 students.

## METHOD

Quasi-experimental pre-test- post-test non-equivalent group design was used in the study. The symbolic representation of the design of the Experimental phase of the study is given below

$$G_1 \; O_1 \; X \; O_2$$
$$G_2 \; O_3 \; C \; O_4$$

Where, $O_1$, $O_3$ – Pre-tests; $O_2$, $O_4$ –Post-tests; $G_1$- Experimental group; $G_2$ -Control Group; X-application of experimental treatment; C- application of control treatment

### Participants

Two intact class divisions of 114 students (60 in experimental group and 54 in control group) of grade eleven students doing science courses from two Government Higher Secondary Schools (Government Higher Secondary School Cheemeni and Government Higher Secondary School Vellur from Kasargod and Kannur districts in Kerala respectively) of rural background following Kerala state syllabus, were selected as the participants.

### Research Instruments

The following standardized tests were used for the present study:

1. Standard Progressive Matrices (1996 Edition, prepared by Raven, Court and Raven published by Oxford Psychologists Press, Lambowne House, Oxford, UK): This nonverbal test is intended to measure the subjects' ability to discern and utilize a logical relationship presented by nonverbal materials.

2. Logical Mathematical Intelligence Test (prepared by the investigators): To measure the Logical Mathematical Intelligence of the subjects, a test was developed based on Logical Mathematical components of the theory of multiple intelligence proposed by Gardner (1983). Reliability of the test was established by the test retest method, on 31 grade 11 students doing science courses. The test-retest reliability coefficient was 0.74. The validity of the test was estimated empirically by comparing the scores of the test with Raven's standard progressive Matrices on a group of 48 grade 11 science students. The coefficient of correlation so obtained was 0.59.

3. Problem Solving Ability Test (prepared by the investigators): This test contained 14 Physics problems from the topics 'Work, Energy and Power'. Reliability of the test was established by the test-retest

method on 54 grade 11 students doing science courses. The test-retest reliability coefficient was found to be 0.77. Content validity was ensured by obtaining the judgment of four experienced higher secondary school physics teachers and three physics teachers in collegiate education from Kasargod and Kannur districts of government and aided sectors. Concurrent validity was estimated empirically by correlating the test scores of 54 grade 11 science students with their scores of Problem Solving Ability test developed by Praveen (2017). The coefficient of correlation so obtained was 0.64. Intervention study was performed to confirm construct validity (Brown, 1996) of Problem Solving Ability test.

All tests were administered as paper-pencil tests with appropriate time restriction.

**Statistical Technique**

Since the experiment was conducted using intact study groups of students of grade eleven, it was suspected that differences in Nonverbal Intelligence and Logical Mathematical Intelligence among subjects would influence the relation between instructional strategy and Problem Solving Ability in Physics. In the present study ANCOVA was used to remove statistically the effects of the extraneous cognitive variables (Non Verbal Intelligence and Logical Mathematical Intelligence) which would have an effect upon the dependent variable: Problem Solving Ability in Physics.

Data Collection Procedure

The investigator himself taught both the experimental group and control groups. The experimental group was taught through Schema Based Instruction. The students were taught the content of chapter 'Work, Energy and power' in the usual expository method of teaching and the problems were dealt in the Schema based method of instruction. The way of teaching Physics problems in the schema based instruction followed the same phases described in the design of schema based instruction. The control group was taught using the Direct Translation Strategy of teaching problem solving. The investigator himself taught the theory and problems of the content portion. The students were taught the content portion of the chapter Work, Energy and Power followed by the worked out problems. The very same set of problems given to the experimental groups were administered to the control group also; but in the usual way of Direct Translation Strategy of teaching problem solving. This would mean that the interventions in the groups differed only in the pattern of instruction of solving problems whereas the learning experiences employed to teach the subject matter remained the same in all the groups. Also the investigator could do justice to the experimental as well as the control groups by teaching them the very same set of problems for work out as well as for practice. The time taken for the entire treatment session was four weeks for each of the study groups. The standardized test for assessing Problem Solving Ability was re-administered in both groups after the completion of the treatment period.

## RESULTS AND DISCUSSION

**Effect of Schema Based Instruction on problem Solving Ability in Physics**:

Table 1 gives the basic properties of the dependent variable Problem Solving Ability in Physics for the experimental and control groups.

| Study group | n | Pretest | | Post test | |
|---|---|---|---|---|---|
| | | Mean | SD | Mean | SD |
| Experimental group | 60 | 4.02 | 1.27 | 18.21 | 3.70 |
| Control group | 54 | 3.78 | 1.47 | 11.70 | 2.31 |

**Table 1:** Distribution of Pre-test and Post-test Problem Solving Ability in Physics

Table 1 shows that the mean and standard deviations of the pre-test scores of Problem Solving Ability in Physics among the study groups do not vary too much. But the mean post test score of Problem Solving Ability in Physics varies among the study groups. This is due to the effect of the intervention applied in the groups.

The effect of instructional strategy (Schema Based Instruction/ Direct Translation Strategy of teaching problem solving) with Non-Verbal Intelligence (NVI) and Logical Mathematical Intelligence (LMI) as covariates on Problem Solving Ability in physics for higher secondary school students was tested using one way ANCOVA. The results are presented in Table 2.

| Source | Sum of squares | df | Mean square | F | Sig. | $\eta^2_p$ |
|---|---|---|---|---|---|---|
| NVI | 9.03 | 1 | 9.03 | 1.01 | 0.33 | 0.01 |
| LMI | 57.84 | 1 | 57.84 | 6.49 | 0.02 | 0.04 |
| Instructional strategy | 1134.69 | 1 | 1134.69 | 127.29 | 0.00 | 0.54 |
| Error | 980.58 | 110 | 8.91 | | | |

**Table 2:** Summary of ANCOVA of Problem Solving Ability in Physics for experimental and control groups

From Table 2 it is clear that the covariates - Non Verbal Intelligence and Logical Mathematical Intelligence - have no statistically significant effect on Problem Solving Ability in Physics for experimental and control groups at .01 level [for NVI: $F(1,110) = 1.01$, $p = 0.326$, $\eta^2_p=0.01$; for LMI: $F(1,110) = 6.49$, $p = 0.012$, $\eta^2_p = 0.04$]. There was a significant effect of instructional strategies on Problem Solving Ability in Physics, after controlling for the effect of Non Verbal Intelligence and Logical Mathematical Intelligence at .01 level [$F(1,110)= 127.29$, $p< 0.001$, $\eta^2_p = .54$]. The partial eta squared value indicates the effect size is large.

Comparing the estimated marginal means showed that the experimental group (which received Schema Based Instruction) has the higher mean of post-test scores in Problem Solving ability (M=18.29; CI=[17.50,19.07]); compared to control group, (M = 11.61; CI=[10.78,12.44]). Thus the ANCOVA result reveals that Schema Based Instruction is effective in increasing Problem Solving Ability in Physics compared to the Direct Translation Strategy of teaching problem solving.

## CONCLUSION

In the present study the investigator attempted to validate a Schema Based Instructional strategy for its effectiveness on enhancing Problem Solving Ability in Physics among grade 11 students. It is ascertained that the Schema

Based Instruction significantly increases the Problem Solving Ability in Physics of grade 11 students, compared to the Direct Translation Strategy of teaching problem solving. Therefore the present study suggests the use of Schema Based Instruction for teaching problem solving. If a problem solving learning environment could be prepared by expert teachers using the elements of Schema Based learning, it could benefit the students to increase Problem Solving Ability in physics. This study clearly proves that the Direct Translation Strategy of teaching problem solving is not useful in improving Problem Solving Ability in Physics compared to Schema Based Instruction. Schema Based Instruction takes care of teaching problem solving in a meaningful way.

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# CONSIDERING A CONTRASTING CASES APPROACH TOWARDS DEVELOPING A RELATIONAL UNDERSTANDING OF THE EQUAL SIGN IN EARLY YEARS

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*The reported study compared two instructional approaches for teaching elementary school students the relational meaning of the equal sign. The primary goal of the larger study, from which the data in this paper are reported, was to examine whether instruction that involves comparing the equal sign with contrasting relational symbols (the greater than– and the less than–signs) is more effective at fostering a relational view of the equal than instruction that focuses on the equal sign alone. Preliminary findings indicate the promise of using a contrasting cases instructional approach to promote appropriate learning about the ubiquitous equal sign in second grade.*

## THEORETICAL FRAMEWORKS AND SIGNIFICANCE

The theoretical underpinnings of this study are rooted in two areas of research in mathematics education: research focused on developing an appropriate understanding of the equal sign and research on contrasting cases approach to the teaching and learning of mathematics.

### Understanding the equal sign in elementary school

Developing an understanding of mathematical equivalence is foundational to success in algebra (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015; Knuth, Stephen, McNeil, & Alibali, 2006). Unfortunately, years of research indicate that the equal sign remains largely misunderstood by young children (Behr, Erlwanger & Nichols, 1980; Carpenter, Franke, & Levi, 2003; Hattikudur & Alibali, 2010), with most elementary school students perceiving it as a request to compute (operational view) rather than a symbol indicating an equivalence relationship (relational view). For example, in equivalence problems with operation on both sides, such as $2 + 3 + 4 = \_\_ + 4$, children typically place either a 9 (adding addends on the left side) or 13 (adding all the addends on both the sides) in the blank. The relational understanding is essential not only for competence in arithmetic problems, but also for success with algebra (Carpenter, Franke, & Levi, 2003; Knuth, Stephens, McNeil, & Alibali, 2006), with an earlier developed understanding leading to better competence in algebra in later grades. It is also essential for success with non-standard arithmetic problems where the unknown appears in a place other than after the equal sign (e.g., $2 + \_ = 4$). As such, overcoming the misconception of the equal sign as an operational symbol early, before it becomes resistant to change (McNeil, 2014), is a crucial educational goal.

In fact, the Common Core Standard for Mathematics identifies that students should be able to interpret the meaning of the equal sign by the end of first grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Given this goal, the absence of explicit attention to the meaning of the equal sign in popular U.S. curriculum is concerning (Powell, 2012). The problem is exacerbated, as many preservice teacher textbooks offer little explanation about how to teach students about the equal sign (Li, Ding, Capraro, & Capraro, 2008). Furthermore, students are typically introduced to equal sign and symbols of inequality in the same grade. Thus, comparing and contrasting the equal sign to symbols of inequality may help in making sense of these symbols as relational in nature.

**Contrasting cases to support mathematics learning**

Researchers have used comparison of solution methods to promote understanding of abstract mathematical ideas (Rittle-Johnson & Star, 2007; 2011). While comparison of solution methods has been used for almost a decade in classrooms, research in recent years has evidenced promising effects of using comparison towards developing students' understanding of concepts underlying abstract mathematical symbols (Aqazade, Bofferding, and Farmer, 2016; Aqazade, Bofferding, and Farmer, 2017). For example, Aqazade, Bofferding, and Farmer (2017) found that second graders' who contrasted the cases of adding two positive integers and adding one negative and one positive integer were able to change their understanding of negatives in addition problems. With respect to the equal sign, there is evidence to support that contrasting the equal sign with symbols of inequality can help in shifting students' view of the equal sign from an operational to a relational symbol (Hattikudur & Alibali, 2010). The researchers found that third- and fourth- grade students who received instruction about comparing the three symbols made greater gains on conceptual tasks (that involved seeing the equal sign relationally) than those who received instruction on equal sign alone.

This study was motivated by the promising findings of Hattikudur and Alibali's work and was aimed at exploring the effects of using such an approach in early grades. The primary goal of the larger study was to examine whether comparing the equal sign with its contrasting symbols (the inequalities) is advantageous over explicit focus on the equal sign alone among first- and second-grade students who are arguably less entrenched in operational view of the equal sign (McNeil, 2014). In this report, preliminary findings on the effects of using comparison on second grade students' understanding of the equal sign as measured by their performances in solving equivalence problems is discussed.

## RESEARCH QUESTION

Does instruction involving the use of a contrasting cases approach: specifically, comparing and contrasting the equal sign (as a symbol of quantitative equality), with the greater than- and the less than- symbols (as symbols of inequality), promote learning any differently than instruction without a contrasting cases approach in second-grade students?

## METHOD

### Participants

Participants (n=36) were recruited from two second-grade classrooms in one public elementary school serving a small mid-western community in the U.S. Participation was voluntary and required parent and student permission. Approximately 72% were eligible for free or reduced lunch and 49% were females.

**Procedure**

Students from each classroom were randomly assigned to one of the two conditions: (a) the *contrasting cases* group received instruction on comparing the equal sign with the greater than- and the less than- sign; and (b) the *equal sign only* group, that also acted as an active control and received instruction focused on using three separate definitions of the equal sign as a relational symbol. The number and type of problems seen by students in the two conditions were identical. Time spend on activities during sessions and any instruction, where applicable, were comparable across the conditions and were video recorded.

Students participated in a total of three one-on-one sessions. The first session was approximately 55 minutes long, and included: pretest, a short break, a lesson based on the group assignment with a discussion portion where students were introduced to the contrasting symbols or equal sign, and a session-exit test. During the second session, students discussed and identified how the contrasting symbols were similar and different, sorted a set of cards with number statements into a true or a false pile (e.g., sorting whether 2 + 5 = 1 + 6 belongs to the true or false pile) and reasoned why they identified the statement to be true or false. The third session was similar to the second session and ended with a posttest that included problems similar to pretest as well as transfer problems.

## PRELIMINARY RESULTS

The outcome measures reported here consisted of students' performance on solving equations, in particular on solving equivalence problems with operations on both sides and one unknown, as well as the reasoning they used to arrive at their answer. Responses were coded for both correctness and the underlying way of reasoning the child used. The coding scheme for students' reasoning was based on a scheme used in prior research (Knuth et al., 2005; Hattikudur & Alibali, 2010). Out of 36 students, one did not complete all three sessions and was excluded from the study.

At pretest, with the exception of two students, all students got zero correct on equivalence problems. This is consistent with previous research (McNeil et al., 2012), which indicates that children typically fail on equivalence tasks, and this has been attributed to children having an operational view of the equal sign (McNeil & Alibali, 2005). Indeed, the majority of the students (98%) indicated an operational view, in their strategy choice by either combining the addends on the left side of the equal sign or adding all the addends when asked to describe how they figured out what the unknown on the pretest.

At posttest, Levene's test for homogeneity of variances revealed unequal variances and hence a one-way-Welch ANOVA was used. The analysis revealed that there were statistically significant differences in posttest scores across the instructional groups, Welch's $F$ (2, 38. 31) = 4.79, $p$ = .007. A Games-Howell post-hoc comparison (at $p < .05$) indicated that the students in the comparison condition (m = 4.68, s.d. = 3.67) performed marginally significantly different than students in the equal sign only condition (m = 3.10, s.d. = 3.32).

**Identifying patterns in performance on equivalence problems**

It must be noted that almost all students had zero correct on the pretest. At the posttest, the students ranged from getting some problems correct to getting all correct, with the bulk of students getting either all or all but one problem correct. A smaller number of students got some but not all problems correct, and thus present an interesting case. Initial interpretations of these students' performance suggest that these students might have only begun to consider the equal sign as something other than an operator but have not yet undergone a complete conceptual change in their understanding sufficient to demonstrate consistent performance on all equivalence problems.

To capture variations in student thinking and performance, students' overall performance was categorized into binary levels; with levels of learning defined as a) *satisfactory*, getting at least four problems correct (out of 6) with at-least one novel problem correct; b) *none*, getting none or just one of the posttest problems correct. These criteria allowed examining patterns of learning that may be associated with the different instructional conditions.

Overall, 62% of the participants' demonstrated *satisfactory* learning on the posttest and 21% demonstrated *no* learning. The percentage of participants by instructional condition with levels of learning is provided in Table 1. As can be seen, the percentage of satisfactory learners varied by instructional group, in the following descending order: *contrasting cases* group (54%), and *equals only* group (48%).

Differences in participants' learning as measured by number correct on the posttest was found to be related to the type of instructional condition, $\chi^2(2,36) = 12.15$, $p = .005$.

## SUMMARY

The preliminary findings reported in this paper indicate that leveraging a contrasting cases approach to instruction might be useful in promoting a relational understanding of the equal with second graders and may help in circumventing the erroneous operational patterns commonly observed among young learners when solving equivalence problems. Further analysis will reveal how students progressed in their thinking and what aspects within the comparison seemed to bootstrap the notion of equal sign as a relational symbol among students on other equivalence tasks.

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# CONCEPT IMAGES OF QUADRILATERALS: A COMPARATIVE STUDY OF VIII AND IX GRADE SCHOOL STUDENTS

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*The study attempts to explore the concept images of students regarding quadrilaterals. The analysis is based upon the theoretical frameworks of Fichebein theory of figural concepts, mathematical relationship and classification of quadrilaterals. Findings of the study suggest that in spite of teaching quadrilaterals with a single definition; students came up with variety of definitions which were personal rather than formal. Students used non- critical attributes in non- formal or incorrect definitions. Trapezium and kite received maximum variations in its definitions and images. Findings of the study suggest that hierarchical classifications of quadrilaterals demand attention in school curriculum.*

## INTRODUCTION

'Quadrilateral' has been described in many ways in mathematics. It is used synonymously for 'Quadrangle' and comes from the Latin *quadric, a combining* form for 'four' and *latus* means sides. 'Quadrilateral' *is defined as 'consisting of four lines, no three of which are concurrent and the six points they determine' (Usiskin and Griffin, 2008)*. Due to the etymology of 'Quadrilateral', it has also been considered to be a polygon with four side as found in many textbooks. Quadrilaterals are formally introduced with its naming at elementary level, first in class VI under the heading of 'basic geometrical shapes' and then in class VIII (named as 'understanding quadrilaterals') and class IX (named as 'Quadrilaterals'). The level of sophistication increases from class VI to IX i.e. from empirical to more axiomatic in nature.

There are number of studies specific to quadrilaterals which built upon Van Hiele levels and in relation to other cognitive aspects. Nakahara (1995) found that an understanding of basic quadrilaterals develops accordance with Van Hiele levels, but it is specific to the geometric figure involved, i.e. level assigned to students may be different for different geometrical figures. Further Transition from level 2 to level 3 has been considered slow and problematic by a number of studies because of the distinguishing feature of level 2 i.e. class inclusion- 'interrelationship between two sets when all the members of the first are members of the second e.g. squares is a subset of rectangle' (Owens & Outhred, 2006) whereas identification of features of a shape and grouping figures based upon a single property are important aspects of level 2 (Clements and Battista,1991; De Villier, 1998;Currie and Peg, 1998; Fujita and Jones,2007). Matsuo (1993) suggested that students' consideration of a square as rectangle depends upon the property they are focused on. If their concept definition consisted of a rectangle having four right angles or parallel sides, there is feasibility that students will consider square as a rectangle and if they consider rectangle with two longer and two shorter sides, square is excluded**.** Researchers aspired to help students overcome difficulties in inclusion tasks, where there is a potential gap between students' geometrical thinking level (Van Hiele, 1986) and task level. For example, the classification of a rectangle as a parallelogram requires Van Hiele's level 3 geometrical thinking (Order), hence students who fail to classify it correctly do not seem to have reached this level (Gal & Lew, 2008). Studies have highlighted the importance of defining and hierarchical classification of quadrilaterals (e.g. De Villier, 1998, 2008; Fujita and Jones, 2007) and argued that formal definitions can be developed at level 3 as at this level, it is expected from students to see the interrelationship between shapes. He has also discussed in his studies that students should be allowed to form visual, uneconomical and economical definitions and should be actively involved in formulations and evaluating definitions. Other studies (Shir & Zaslavsky, 2002; Heinze, 2002; Saenz-Ludlow & Athanasopoulou, 2007; Villier, Govender & Patterson, 2009; Blair and Canada, 2009; Driscoll et.al., 2009) focused on active engagement of students in defining processes such as to analyse definitions; create and critique their own definitions; reason with relationships. These studies have shown that students who are encouraged to participate in defining are able to change their opinions because of interactions as well as justifying and arguing with their colleagues and it leads them to positive gain in understanding nature of definitions and also to think about necessary and sufficient conditions of definitions. Fujita and Jones have emphasized the need of a theoretical framework for the development of definitions and hierarchical classification of quadrilaterals. They argue that a study of hierarchical classification can help in bridging the gap between Van Hiele level 2 and level 3. They also proposed to explore the common cognitive paths of the relationship among quadrilaterals as from their study it was speculated that there could be hierarchical order of the difficulties. For example, they conjectured from their study that it might not be effective to teach the relationship of rectangles and parallelograms before the relationship between rhombuses and parallelograms (Fujita & Jones, 2007).

## THE PRESENT STUDY

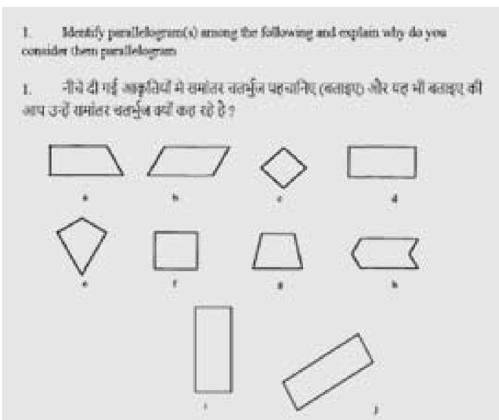
The objective of this paper is to put forth the concept images of elementary and secondary school students regarding quadrilaterals. This research is in consonance with the assertion that Villiers et.al. (2009) states as 'The classification of any set of concepts implicitly or explicitly involves defining the concepts involved, whereas defining concepts in a certain way automatically involves their classification'. Definition and classification are, therefore, important tools for developing the ability of deductive reasoning and proving. It also plays an important role in identifying new mathematical objects with some precision. However, a number of studies reveal that many learners have difficulties with hierarchical classification of quadrilaterals and related formal defining of such shapes because of cognitive complexities involved in such learning (e.g. Monaghan, 2000; De Villier, 1994).This study considers the complex nature of 'figural concept' as a major factor of learners' difficulties with hierarchical classification of quadrilaterals and related formal defining as also studied by Fujita & Jones, (2007). Fischbein states that 'while a geometrical figure (such as a square) can be described as having intrinsic conceptual properties (in that it is controlled by geometrical theory), it is not solely a concept; it is also an image' (Fischbein, 1993, p. 141) implying that a geometrical figure has characteristics of dual nature in that it is both concept and image and the two are closely interrelated. Learners lack the ability to combine interaction between a concept and its image and hence individuals' *personal figural concepts* are formed that *influences* the classification of quadrilaterals. Personal figural concepts are formed on the basis of and individuals' *personal geometric concept definitions* that are shaped by the *concept images* formed in their minds and which are different from *formal conceptual definition* of a geometrical object.

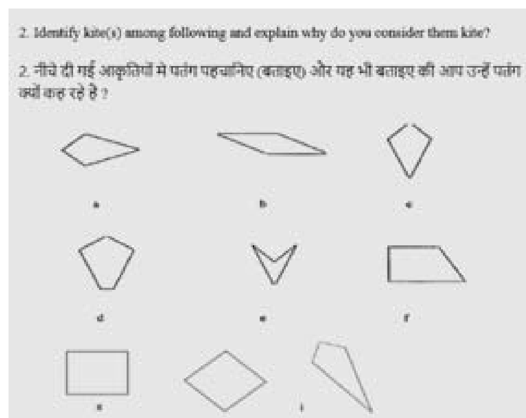
### Participants and Setting

The study group consisted of 240 students from 4 sections, each of classes VIII and IX (having 30-35 students in each section) from the two government schools for using worksheets and students' interviews. The government schools had strict admission criteria and had both English, Hindi meduim. However, in spite of having English medium, meduim of instructions commonly used was Hindi. Both English medium and Hindi medium sections were selected for the study. The socio-economic background of students varied from working to lower middle class. Parents' occupations varied from worker in electricity board to, drivers, teachers, farmers, small business to vender etc.

### Data Resources

After the topics were taught in the classes by the respective teachers, worksheets related to geometrical tasks were given to students to assess students' understanding of quadrilaterals. The worksheets included items related to (i) Identification of different quadrilaterals and (ii) Defining quadrilaterals.




1. a. Having experiencing from your surroundings and learning from different resources like textbooks, your school or tuition teachers, your friends write down your Ideal definition of the following shapes in your own words:

(i)Polygon, (ii) Quadrilateral, (iii) Parallelogram, (iv) Rectangle, (v) Trapezium (vi) Rhombus, (vii) Square (viii) Kite

b. Also show them diagrammatically
c. Draw and name one example and non-example of these shapes

1. a. आपके आसपास के अनुभव से अपनी पाठ्यपुस्तक, स्कूल अध्यापिका, ट्यूशन अध्यापिका, आपके दोस्त इत्यादि से बातचीत करके या सीख कर, आप नीचे दी गई आकृतियों कि अपने शब्दों में अपनी अभीष्ट परिभाषा दीजिए i) बहुभुज ii) चतुर्भुज iii) समांतर iv) चतुर्भुज v) आयात vi) समलंब vii) समचतुर्भुज vii) वर्ग viii) पतंग

b. इन्हें चित्र द्वारा अंकित जी कीजिए |
c. इन आकृतियों के लिए एक उदाहरण व एक अउदाहरण (जो उदाहरण ना हो) दीजिए |

**Figure 1:** Worksheets (i) and (ii)

**Data Analysis**

Data collected through worksheets and interviews was analysed through the process of identifying, coding and categorizing the primary patterns in the data. In the study, students definitions, drawings and reasoning while identifying quadrilaterals were taken into account while analysing students' perceptions. Results were presented in the form of frequencies, students verbatim and their drawings.

**Key terms used for the purpose of analysis are as follows:**

- Partitional and Hierarchical classification: Partitional classification includes exclusive definitions, which consider concepts involved as disjoint from each other e.g. squares are not considered as rectangle. Hierarchical classification includes inclusive definitions which allow inclusion of more particular concepts as subsets of general concepts e.g. square is classified as a special rectangle
- Formal concept definition: formal is related to mathematically accepted; a concept definition is defined as 'a form of words use to specify that concept' (Tall and Vinner, 1981, p. 152).
- Personal concept definition: It is defined as students' own definitions based upon their own experiences of learning geometry and are different from formal.
- Concept Images: 'The total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes' (Tall and Vinner, 1981, p. 152).
- Formal figural concept: A 'figural concept' which is associated with formal definitions and formal concept images in geometry.
- Personal Figural concept: A 'figure concept' which is associated with personal concept definitions and one's own concept images in geometry.
- Critical attributes are attributes that must be present for the concept to be formed, while non-critical attributes are attributes that may be present but are not required and has strong visual characterstics.

## RESULT AND DISCUSSION

In this section, a comparison of concept images of VIII and IX grade students is provided in relation to quadrilaterals. It has been presented in the light of definitions and drawings and reasoning provided by the participants.

**Definitions and Drawings of quadrilaterals of VIII and IX grade students**

Definitions and drawings of 'basic quadrilaterals' were analysed to explore the students' personal figure concept. Comparison of the images and definitions of 'basic parallelograms' displayed in table 1.1 shows that the majority of students of classes VIII and IX could draw a prototype image (correct image) of quadrilaterals (with an exception of trapezium), however very less students provided their respective correct definitions. Least correct definitions were provided for 'trapezium' and 'kite'. Interestingly, there was not a significant difference in the frequency of respective responses of class VIII and IX.

| Class | Class VIII (%) N=120 | | Class IX (%) N=120 | |
|---|---|---|---|---|
| **Figure** | **Image** | **Definition** | **Image** | **Definition** |
| Parallelogram | 98 | 30 | 99 | 31 |
| Rectangle | 100 | 33 | 98 | 33 |
| Trapezoid | 71 | 26 | 86 | 29 |
| Rhombus | 100 | 32 | 99 | 34 |
| Square | 100 | 33 | 100 | 33 |
| Kite | 93 | 24 | 98 | 28 |

**Table 1:** Correct responses of drawing and defining of 'basic quadrilaterals'.

When students' definitions of different quadrilaterals were examined, a variety of definitions were observed ranging from listing critical attributes to describing a figure by its name of shape using non- critical attributes. Maximum variations were observed in the definitions of *parallelogram* and *trapezium.* Interestingly, critical attributes were used for defining parallelogram and non- critical attributes were used for trapezium eg. Trapezium as: *A closed figure made up of two figure(s), a triangle and a quadrilateral*, Student H, class 9). Most of the non- formal (natural speech) but satisfying minimal necessary and sufficient conditions were observed for 'kite'. eg. Definition of kite provided by student G of class IX *Upper two lines are equal and lower two lines which meet each other are also equal.* In this definition, he wanted to explain that adjacent sides are equal.

Analysis of definitions also revealed that 84% students of grade VIII and 48% students of grade IX provided definitions having unnecessary or insufficient conditions required for constructing their respective figures. Examples of students' definitions (interpreted) containing unnecessary conditions are (i) *Quadrilateral: It is a four-sided closed figure, it has no properties of sides and angles. (ii) Trapezium: A line which stands on the other line is called trapezium.* An example of definition consisted of insufficient conditions is as follows: *Rectangle: Rectangle is a parallelogram but parallelogram is not a rectangle.*

This definition is insufficient in the sense that, it tells that the rectangle has all the properties of parallelogram but conditions which makes it rectangle was not mentioned in this definition. It was also seen that among rest of the correct definitions provided by students of both grades, most of the definitions contained more than minimal set of necessary and sufficient conditions, termed as *uneconomical* (Villier. et. al. 2009). An example of uneconomical definition shared in natural language by a student of class VIII is: *A figure which has four sides and it is closed from all sides. its opposite sides are parallel. Sum of all angles is 360°*.This definition contains more properties than required to construct a parallelogram.

Some common factors emerged that influences student definitions are categorized as follows:

**Language driven**

It was interesting to see that 17% of class VIII and 20% of class IX students' definitions of trapezium were influenced by **its name in Hindi language** 'samlamb' (sam means equal, lamb means perpendicular). Another figure in which it was expected to have the influence of language, i.e. parallelogram as stated by Fujita and

Jones (2007) that students may tend to remember about parallel lines from its name 'parallelogram' but they may have limited perception /understanding of it, definitions of parallelograms were not much influenced by its language. Despite of the fact that the name of parallelogram, may reminds students about parallel lines, 22% of class VIII and 25% of class IX students used criteria of 'opposite sides are equal'. Interestingly, criteria of 'parallelness' were used by more number of students of class VIII (23%) than class IX (13%). Many students used linguistic explanation for parallel as equal distance between two lines or lines never bisect (interpreted meaning intersect). In cognitive sense, definition may be influenced by its name but its concept images may be limited. For e.g. rectangle, rhombus and square are termed as special parallelograms under the hierarchical classification of quadrilaterals; limited perception about parallelogram may not consider them as parallelograms.

**Non-critical attributes**

To define trapezium, 15% of class VIII students and 12% of class IX students used **non-critical attributes** of its prototypical image e.g.

- Trapezium is made up from two triangles and one rectangle
- One triangle and one rectangle
- Look likes rectangle, but sides are not equal
- Looks like a table or pot
- It has parallel sides and perpendicular height.

Some typical parallelogram's definitions of class VIII students (20%) were influenced by its prototypical image like 'opposite sides equal and bends' or 'opposite angles are equal and angles are more than 90°'.20% of class VIII students and 20% of class IX students used non- critical attributes to define kite. Some typical definitions stated by students are:

- *A figure whose all sides are equal, but it is not a square.*
- When we see it, it looks like a prism?
- (Diamond) is called a kite.

**Critical attributes and economical definitions**

- Most of the definitions of parallelogram were economical definitions using *equal side criteria*.
- Only 18% students of IX class and 7% students of class VIII defined trapezium as a quadrilateral with *one pair of opposite sides parallel*.
- Definitions of rectangle and square had a strong influence of its prototypical image. Therefore, in both the definition, students did not feel to mention about its angles with other critical attributes of their respective images. Angle is an important attribute of rectangle and square as it differentiates parallelogram from rectangle and rhombus from square. Table 2 shows that percentage of students who defined square and rectangles without mentioning about its angles.

| Class | Square- all sides equal | Rectangle-opposite sides equal |
|---|---|---|
| VIII | 38% | 43% |
| IX | 25% | 15% |

**Table 2:** Students' definitions of square and rectangle

- In case of ‘kite’, maximum number of economical definitions were written in students’ natural speech’ i.e. non- formal definitions, for example, non-formal definitions to explain adjacent sides equal by student G of class IX ‘*Upper two lines are equal and lower two lines which meet each other are also equal’*.

Analysis of students’ drawings indicates that most of the students drew prototypical image of these concepts (except trapezium). It was emerged that students used three different ways of connecting an image with its definition:

- Influence of image on its definition
- Influence of definition on its image
- Influence of name of the figure on its definition and hence on image

**Influence of image on its definition**

The responses of 31% students of class VIII and 17% students of class IX described quadrilaterals without mentioning whether it is a quadrilateral/a four-sided figure/a simple closed curve. The image has such a strong influence that they did not consider it important to mention this while describing its attributes or properties. Student A e.g. gives the following response when asked to define a square *All its sides are equal, opposite sides are parallel, all sides are equal to each other also. All angles are of 90°. Diagonals bisect each other.* The student, however fails to mention that a square is a kind of quadrilateral or a parallelogram or any other shape. Emphasis on ‘**its’** shows that the student is focused on the image drawn by him/her. In another example of defining a ‘kite’, Student B responded that *‘Kite is shaped like a rhombus when we rotate it”*. In the definition of a rhombus, Student C replied that *all its sides are equal, but its shape is different from a square*. Influence of prototypical image is so strong that the learners feel the need to mention in the definition that it is different from a square. This definition would fall in the category of a partitional classification, i.e. in that it excludes square from the set of rhombus (Villier et.al., 2009). It may be interpreted that concept image of a definition may be responsible for students’ exclusive definitions. This connection of image with its definition is has also been borne in the findings of Fujita and Jones (2007).

It was also observed that definitions in natural speech (non -formal) had influence of non-critical attributes of the prototypical images. For example, it was observed in definitions of 33.33 % of class VIII students and 40% of class IX students e.g.  Student E defined kite as “*Kite is a shape which has two triangles together and those two triangles are not equal”. Another Student F defined kite as* **“***both its vertices are outside, on the top and at the bottom”*. (Here learner wanted to convey that upper and lower vertices of kite are more pointed than left and right vertices). In both the definitions, learners were describing non-critical attributes (attributes which evoke visual image of shape) of kite in its definitions. In the case of the trapezium (42% of class VIII and 30% of class IX) and kite, definitions using non-critical attributes seem to indicate an especially compelling influence on the image. So, for example a student who defines the trapezoid as: *‘A closed figure made up of two figure(s), a triangle and a quadrilateral’ represents it as in the picture below: (Student H; Class 9)*

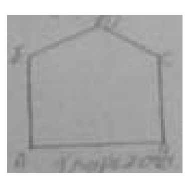


**Figure 2:** Sample of student’s drawing of trapezium

A prototypical image of a trapezium can be decomposed into a triangle and a quadrilateral by joining a vertex with any point on its opposite side. This Student has visualized the decomposed shapes and recomposed it into a pentagon.

**Influence of definition on its image**

The results also indicate that the definition has influenced the image. There were some typical cases where this connection was clearly visible. For example, Student J defined rhombus as*: A four-sided closed figure with 90°angle is called rhombus. Each side is equal.*

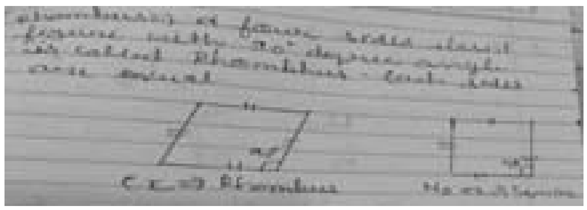


**Figure 3:** Sample of student's drawing of rhombus

Since the student defined the rhombus consisting of 90° angles, therefore he/she labels the obtuse angles in the figure as 90 degree angles. This connection between image and definition may work if students try to memorize the definition of figure that may lead to mix up properties of some other figure e.g. properties of square in this case are mixed up with properties of rhombus.

**Influence of name of the figure on its definition and hence on image**

The results also suggest that the name of the figure affects the ways in which students define the figure and also the kind of mental image they created of it. This was mainly observed while defining trapezium by 17% of class VIII students and 20% of class IX students. The trapezoid is called '*Samlamb*' in Hindi, and its literal meaning would be 'equal perpendicular'. Some interesting examples when asked to define a trapezoid student response were as follows: (translated from Hindi)

- *A figure in which one line is perpendicular to the second line.*
- *A line which stands on the other line is called 'Samlamb*

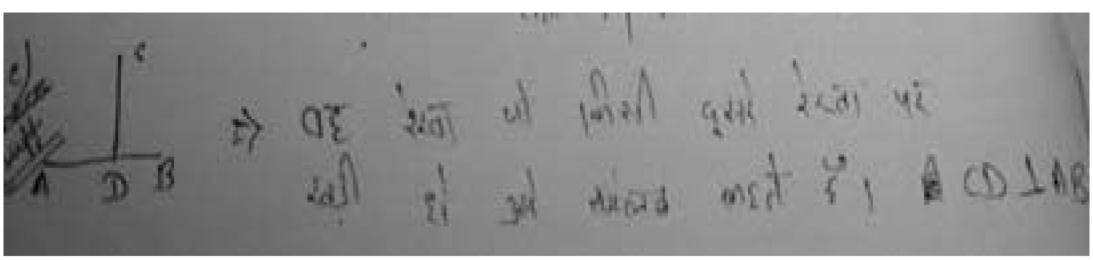


**Figure 4:** Sample of student's drawing of trapezium

Another example is:

- *Trapezium (Samlamb): A four-sided figure, with four equal sides and all sides make 90° angle, is called trapezium*

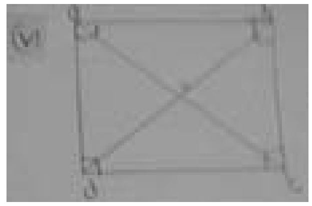


**Figure 5:** Sample of student's drawing of trapezium

## CONCLUSIONS

Data of the study revealed that the students predominately used personal figural concept. There was a considerable gap between formal figural concept and personal figural concept of both classes VIII and IX. These findings are consistent with the finding of research done by Fujita and Jones (2007) ('kite' was not studied by them) and Kawasaki (1992) who studied teacher trainees' personal figural concepts of quadrilateral and found a significant gap between formal *figural* and *personal figural concept* of quadrilaterals. The study suggests interrelationship of quadrilaterals needs to be focussed more in school curriculum.

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# DESIGNING AND MAKING ROLLER COASTERS BY INDIAN MIDDLE SCHOOL STUDENTS

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*The current article details our observations from two workshops with grade 9 students, working in groups, designing and making a paper roller coaster. The preliminary analysis of our observations indicate that designing an activity around roller coasters has huge potential in terms of giving students first-hand experience of designing, open-ended thinking, exploration and modelling. The activity enabled communication and collaboration between members, engaged them in planning, designing, model-making and executing design ideas.*

**Keywords:** *design education, modeling, making, roller coasters, collaboration*

## INTRODUCTION

Maker-centred learning provides opportunities to learners to acquire both critical and creative thinking skills through creating, designing, making and tinkering activities. These activities promote active participation, self-directed learning and encourage taking up of challenges as creative learning opportunities (Clapp, Ross, Ryan & Tishman, 2016). In the context of design problem-solving, a typical maker-centred activity would involve ideation, material exploration and manipulation, mock-ups, model-making, and prototyping. Studies with practising designers have shown frequent use of mock-ups and rapid prototyping (Hess & Summers 2013; Deininger, Daly, Sienko, & Lee, 2017) at different stages of design process to benefit the final outcome. Amongst children, the process of model-making has also been reported to aid in externalising and verbalising ideas that might otherwise be difficult to communicate (Yrjönsuuri, Kangas, Hakkarainen, & Seitamaa-Hakkarainen, 2019).

### Context of this study

Roller coasters (RC) are fun rides to be on. They have often been the object of interest for children, even when they have never experienced riding on one. Making and designing RCs is not new to educators (Cook, Bush & Cox, 2017; Jones & Jones, 1995; Ansberry & Morgan, 2008). One finds a variety of activities on science (typically concepts of potential and kinetic energy, velocity/speed, gravity and laws of motion) and engineering design surrounding RCs. However, in the process of implementing such activities in a classroom setting, learners tend to concentrate on the procedural aspects of making their designs, instead of exploring relevant science content and design principles. Vattam & Kolodner (2008) refer to this tendency as the "design–science gap". One may also argue, that procedural prototyping (for example template based or ready-to-use kits) may not be ideal for children who are novices in designing as it might not facilitate learning and mastering fundamental design and engineering skills such as measurement, precision, estimation and approximation (Choksi, Chunawala & Natarajan, 2006).

Although, activities pertaining to designing and making RCs are quite popular in school science education, they are not documented with Indian students, especially from a design perspective. In this exploratory study, we share our observations from two design and technology (D&T) workshops which were conducted with the aim of introducing students to the iterative design process from conceptualisation to making and evaluating.

## METHODOLOGY

Two stand-alone workshops of approximately 3.5 hours each were conducted a day apart with grade 9 students. The workshops were planned with a focus on engaging students with design exploration, planning and making skills, collaboration, communication and evaluation.

### Participants and Data Collection

The workshops involved Grade 9 students (age group 13-14 years) of Jawahar Navodaya Vidyalaya (JNV) schools from 3 Indian states (Gujarat, Maharashtra and Goa). JNV schools are central government residential schools spread across India, where the medium of instruction is both Hindi and English. There were two batches of students. One batch had 29 (8G, 21B) students and the other 31 students (19G, 12B). Students were from rural, semi-rural and urban backgrounds and many were unfamiliar with others in the batch. Student groups were formed by a random chit-based system. Most of the groups happened to be mixed gender groups with 4-5 students in each group. In total, there were 12 groups which have been named as G1, G2, G3, and so on, in this paper.

*Data sources*: Field logs from 4 facilitators, informal and formal interactions with students, students' drawings, the RC models made by students, photographs and videos, student presentations, and the worksheets filled by students of workshop 2, served as sources of data.

### Procedure, Design Brief and Materials

The workshops were planned to engage students on a D&T task in a playful manner. Workshop 2 (W2) was re-structured after reflecting on the learnings of Workshop 1 (W1). Both workshops started with an initial discussion of students' ideas about D&T. This was followed by a brief 10 minutes presentation on D&T with a focus on iterative and collaborative nature of design. Subsequently, students were involved in the following steps: 1) Practising basic skills (paper folding techniques) needed for the RC design challenge; 2) Brainstorming, planning and making rough drawings of their RC design; 3) Building or Making the RC. 4) Testing and evaluating their RC model by using a marble and revising the models. 5) Demonstrating their final working RC model and communicating it to their peers. There were some differences in the ways the 2 workshops (W1 & W2) were conducted. These differences were: initially, information about the design task was unknown in W1, while it was known in W2. In W1, facilitators engaged the whole class together for making different RC tracks, while in W2, three workstations were setup simultaneously to demonstrate the same. Use of funnel and wide loop was mandatory in W2, but not in W1. A worksheet for reflection was given only to students in W2.

> *Design Brief*: *Imagine you are a roller coaster design team competing to design a new and exciting roller coaster ride for a playground. Your task is to design and build a mini paper model of the ride using the paper folds taught to you. You have to test your ride with the marble provided. The specifications are: 1) The entire roller coaster must fit on the base provided; 2) The marble on your ride should travel for at least 4 seconds; 3) You have to use at least 1 pillar, 1 loop, 1 straight track and 1 L-shaped track. Also, 1 wide loop and 1 funnel (specific to workshop 2); 4) It should be a self-supporting model; 5) Marble should travel from start to end without any external interference.*

*Materials provided to students:* Coloured paper, scissors, glue, stapler, sticky tape, pencil, scale, a base box with dimensions: length=45cm and width=50.5cm (Workshop 1) and length=29cm and width=38cm (Workshop 2).

## OBSERVATIONS AND FINDINGS

Our analysis focused on design process, specifically exploration, generation of ideas, modelling and collaborative designing. Design process was analysed using students' sketches, their final RC models, student discussions and conversations as reflected in our field notes, and photo documentation of the workshops. Insights for evolution of ideas were drawn from students' design explorations and negotiations. Collaborative designing was analysed in terms of distribution of responsibilities and use of verbal and non-verbal communication modes.

### Design process and evolution of ideas

Building of a RC as a design challenge was received with a lot of enthusiasm by students. They were focused while learning new paper folding techniques and curious to know how these paper folds can be placed together to make a working RC. After the initial session of demonstration and learning folds, the groups were involved in making a working model, and this required planning and teamwork. The design challenge involved ideation, design and planning, making and testing. D&T education places emphasis on developing technical abilities of students in addition to imparting essential skills like modelling. Stables and Kimbell (2000; 2006) stress the strong interaction between mind and hand during design and making activity, indicating that they are inseparably linked. Harrison (1992) argued that in schools, most making should in fact be modelling. He categorised the purposes of modelling as i) aids in thinking, ii) communicating form or detail, and iii) evaluating a design or selecting its features.

**The process of constructing the roller coaster:** All the teams were encouraged to make a design sketch of their RC first and plan the requirement of parts. However, we observed that only few groups chose to sketch before starting to make the RC. Prior to students making the models, facilitators had demonstrated two different approaches of building a roller coaster. First, making the path with various parts and later give support with pillars and second, start with building a skeleton structure with pillars only and later add tracks to design the RC. The students had the freedom to choose either option as their approach for designing and execution. Most groups began with the second approach but moved on to using a combination of both methods. One of the major constraints that seemed to influence students' design approach was being aware of the limited time in hand. As a result, most of the teams (G1, G2, G3, G6, G8, G9, G10, G11, G12) simultaneously started making a minimum set of pillars, straight tracks and at least one L-shaped track. Only G5 approached the design task with one task at a time, that is, all members made pillars first, followed by making tracks. Once the teams made a few parts ready for use, tasks were distributed amongst members to assemble the RC model. Estimating the number of pillars they would require to build the RC was important and team members spent considerable time making pillars accurately. Making the paper folds especially the loops required skill and teamwork. It involved careful measurement, accurate cutting, folding and sticking the folds together to form a loop. The entire exercise of measuring and joining of parts was essential in holding the entire weight of the RC. Most groups wanted to make their RC such that it would to be able to hold the marble for the longest time. But they struggled with ways to make time extenders. With constant explorations and testing, they were able to overcome these challenges and came up with a variety of ways to extend the time spent by the marble on the RC. Irrespective of the approaches used, we observed that all groups managed to successfully complete the task within the stipulated time (3.5 hours).

**The use of sketches:** In design research, sketching has been reported to play a crucial role in generating, developing and communicating ideas (Goel, 1995). However, children may not consider sketching as useful as it is considered by design professionals (Rogers, 1998) and children may shift to exploring 3-dimensional modelling for prototyping and ideation (Welch, 1998; Rowell, 2002; Hope, 2005). Our observations align with the aforementioned. It seemed that the initial use of 2-dimensional sketches for the RC model was not necessary for children; as some groups (G1, G3, G7, G8, G10, G11) retained only a few parts (like the start and end) from their sketch in the final models. Interestingly, we observed G3 sketched and planned on the base provided to them, in addition to a rough sketch on paper. They marked positions for pillars on the box, estimated the number of pillars required and then started making pillars. Two groups (G9, G12) reported that their final model was completely different from their initial sketch. It is evident from their sketches and respective models that none of final models was the same as the initial design sketch. Our observations indicate that in this making task, students preferred 3-dimensional modelling over sketching for thinking, planning, communicating and developing design ideas.

**Design exploration and negotiating challenges:** Research studies performed with practising designers have shown frequent use of mock-up models and rapid prototyping aids in early identification of design issues, discovering opportunities, conceptual design and eliminating less promising solutions (Hess & Summers 2013; Deininger et al., 2017). Children can also use prototypes as thinking aids for both refining their models and developing design ideas (Yrjönsuuri et. al, 2019). As the students built their RC models, they explored design possibilities, discovered issues with their initial designs and invented novel solutions sometimes through trial-and-error and material manipulations. During the modelling process, they encountered several challenges, such as placement and joining of the parts. At times, the marble would get stuck or would not jump from one track to the other as estimated or it would derail from the track due to excess speed. Through continual revision and testing, however, students were able to evaluate and resolve these problems. Groups spent a considerable amount of time to figure out the size, placement and adaptability of parts such as loops and tracks on the RC model. The procedure to make a funnel or a wide loop had not been demonstrated by the facilitators in workshop 1 (W1). Hence, it was not a part of initial design sketches in any of the groups who participated in W1. But as they progressed with making RCs many students insisted on having funnels and wide loops in their RC model (which they had seen in the demo model). Thus, G10 had initially planned to put one wide loop, instead they placed an S-shaped loop made by combining two wide loops. For G12, the marble did not move through the RC track and got stuck in between. They often had to make adjustments in positioning of pillars, in the wide loop (they had to remove and again stick it properly) to maintain a smooth flow of marble. Therefore, children constantly used their models for exploration, design ideation and evaluation.

Regarding the various technical and design hurdles that students faced, we observed that for the most part students were able to resolve these by themselves without the facilitator's intervention. Students were actively engaged in the task and skilfully made all the different tracks and carefully placed them on the base provided. An example where students did approach the facilitators is described in the case of G3. A boy from G3 showed one of the facilitators a curved loop that the group made and inquired, "I*f I put the marble on this normal loop, then it falls… so, how should I make it (the loop)?*". This inquiry could be a result of knowing the possibility of another type of loop (a wide-loop, which was shown in the demonstrated models designed by the facilitators, but was not taught to students in W1) and the inability to make the same. Here, the process of making, testing, evaluating and not being able to achieve what was desired, made the team members seek our guidance and look for other options. Overall, wide-loop was found to be one of the most difficult parts to make by the students in both the workshops. While making the wide-loop, students got confused between the wall and base of the track and often ended up making a regular loop (U or C shaped) instead of a wide loop (G2, G3, G5, G8, G10). It is only when they tried to stick it to their RC structure, they found mistakes in the making of the wide loop.

G3 also faced a technical glitch and the group members proposed and discussed potential solutions to address it (Figure 1 and Transcript 1 below from field notes). Interestingly, the group discovered that lack of smooth rolling worked in their advantage by increasing the time the marble spent on the track.

Boy 1: The marble is not rolling smoothly from here (gestures by pointing at a location) to here (gestures by pointing at another location).
Girl: Shall we increase the height of this pillar?
Boy 2: But we have already stuck the pillar.
Girl: We need to somehow give it a lift here (pointing at the blue corner)
Boy 3: We can't add pillar here (gesturing at blue corner). [this could be since the design brief insisted on restricting the RC model to the base provided].
Girl: Ok, let's first finish this part (gesturing the track ahead of their problem) and come back to this.

**Transcript 1: Adapted from Field Notes.**

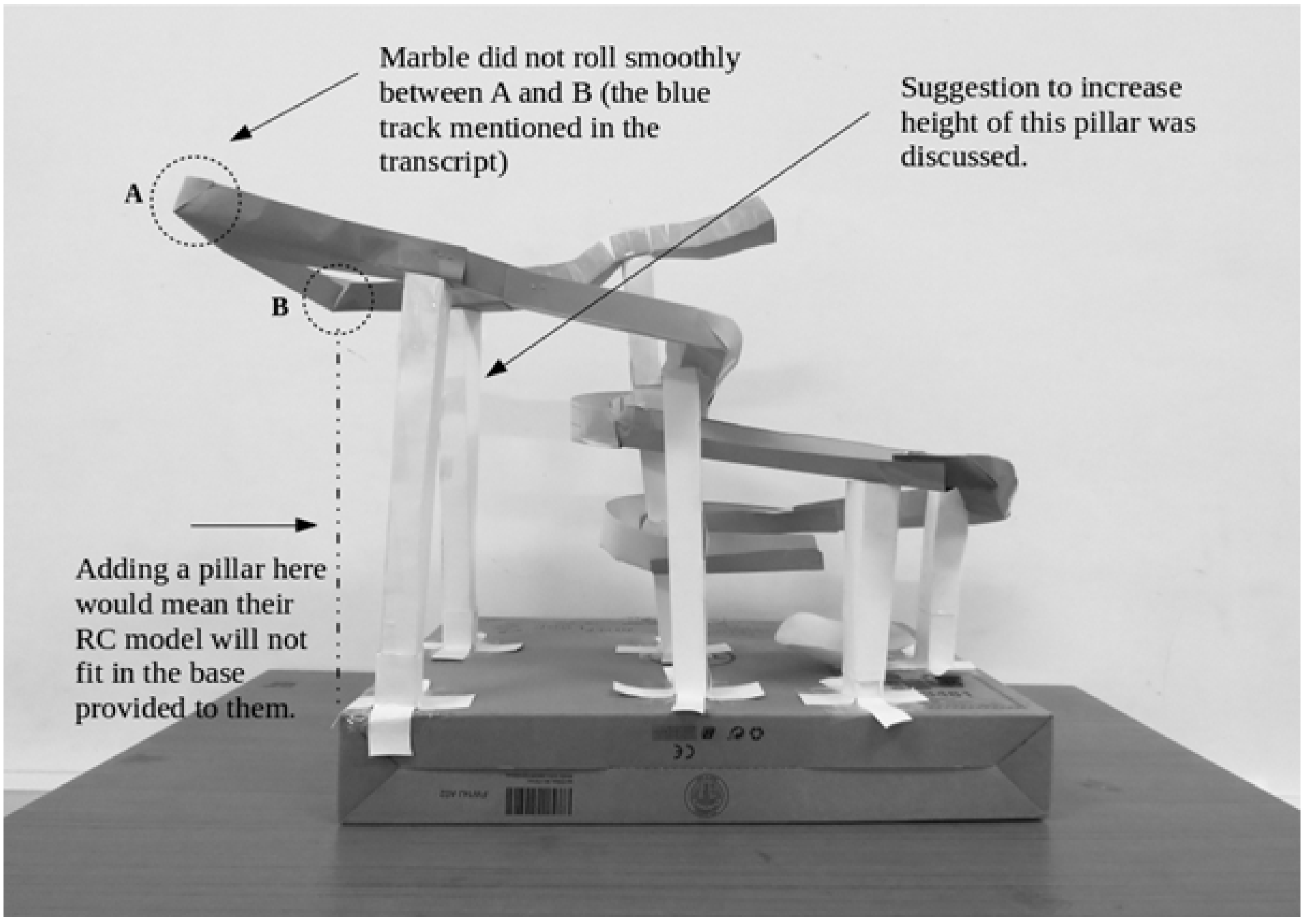


**Figure 1:** Roller coaster model of G3

**Elements of novelty:** Within the context of this workshop and our analysis, student explorations and design ideas were considered 'novel' if the ideas were unique (design and use of new elements which was not demonstrated by the facilitators) within the group of participating students i.e. 'local novelty'. Students brought in various novel elements in their RC model under different circumstances. For example, G1 realised they had missed one of the design brief requirements in the RC, a loop. Rather than redoing or moulding the entire RC, they added an innovative element in their design; a disconnected stand-alone half-arc to capture the marble towards the end. The force with which the marble would fall there, would make it go up the arc for a few centimetres, thus decelerating and then the marble would roll back down the arc into the funnel. This, according to the students, added approximately 1.5 seconds to the time spent by the marble on the RC. Another incidence involved G3 realising that their model had a flaw (see Transcript and Figure 1) in one of the tracks, which they later intentionally retained because it increased the time considerably, the marble spent on the RC. Additionally, G3 also explored making a different type of a pillar, in which they inserted one pillar into the other so as to make it adjustable as per the requirements but decided against using it.

More instances of novel elements include; G7 experimenting with a unique use of the pillar structure to drop the marble and make it an interesting starting point. The ‘uniqueness’ entailed using a pillar as a track. They also made a few hexagonal pillars, and used a track to strengthen another track. G6 made an extended flap which would capture the marble in a compartment. G10 had included a triangular pillar in their model. Some groups also made multiple wide loops (G8, G9, G11, G12) that increased time or introduced twists around the pillar which led to adding levels in the RC. Some models were designed to have intentional breaks in the tracks leading to the marble dropping onto another part (G1, G10, G11, G12). Groups G5, G6, G7 and G9 made RCs which they thought may not keep the marble on for long. Hence, they generated ideas on time extenders and made a variety of speed-breakers (G5, G9), flaps (G9) and multiple staircases (G5, G7, G9). We also saw time extenders being used by other groups (G2, G10). Interestingly, students in G5 and G6 decorated the RC base with the remaining coloured pieces of paper which according to them, added to the aesthetics.

**Modelling and collaborative designing:** During collaborative designing, design ideas become visible for joint evaluation and development through materialisation and model making (Ramduny-Ellis, Dix, Evans, Hare, & Gill, 2010). Prototyping can be used as an effective method to externalise ideas that might otherwise be difficult to imagine, explicate and verbalise (Yrjönsuuri et. al, 2019). In this section, we showcase instances of how a shared goal of co-creating an RC model encouraged collaboration between group members through material handling, and making and assembling parts of the RC model. In this section, we elaborate on two important aspects of collaborative designing. One was the distribution of tasks and responsibilities and the second was the use of verbal and non-verbal modes of communication. The task of making a working RC in a group within time constraints and the material manipulation and assembling required a lot of coordination and teamwork. Lahti, Kangas, Koponen, and Seitamaa-Hakkarainen (2016) have reported linkages between model making and materials with division of labour. Particularly, we observed instances where a student was involved in a making task, and either requested help or was offered help to achieve the task (G5, G11). In most of the groups, leaders emerged implicitly or explicitly, and delegated responsibilities, built a consensus among team members and steered the group closer to their final design.

**Figure 2:** Left- The girl gestures to her group members, the ideal height of the second pillar (G6); Right- Students move the free-track up and down to check for the right incline, before sticking it to the pillar (G11).

The RC model-making acted as a central point of focus which facilitated and mediated discussions through the use of verbal and non-verbal modes of communication. Härkki, Hakkarainen and Seitamaa-Hakkarainen (2018) have noted co-working on building models helps in verbalising ideas, and the role of non-verbal modes of communication, such as sketches and gestures does so as well. They emphasised that gestures play a dynamic role in creating and shaping design ideas leading to further refinement of design ideas. There were numerous instances during the stages of planning, making and presenting, where students resorted to a variety of non-verbal modes of communication, mainly in the form of gestures to propose solutions and communicate problems (Figure 2). Students also used gestures to convey emotions of excitement or disappointment when their model 'worked' or did not.

## DISCUSSION

Though the idea of making paper RCs is not new to educators, one has not seen it in practice in Indian settings nor has this been documented. These workshops offered a glimpse of how such a D&T activity can be planned for a class size of around 30, for a time period of around 3.5 hours. Students engaged in open-ended design problem solving which led to the creation of a working model of RC. All groups successfully fulfilled the requirements mentioned in the design brief by acquiring and strategically using paper folding and modelling skills. In fact, no two models were the same and all the models could hold the marble for more than 5 seconds. Our observations revealed that students used a combination of approaches when constructing the RC and did not adhere to their initial sketches much. They preferred 3D modelling over 2D sketches to ideate and communicate. This is perhaps a difference between the way professional designers and school students approach a design problem. More opportunities thus may be built into the structure of the workshop allowing for material exploration and manipulation.

Previous studies have indicated that students often refine their design over the course of planning and making (Khunyakari, Mehrotra, Natarajan, & Chunawala, 2006). Our observations also indicated that students extensively explored various design options while making and testing, negotiated and overcame challenges, and brought in novel elements to their RC model. Lastly, the challenge of model making, and group work resulted in students using verbal and non-verbal modes of communication, which aid collaboration (Jeong & Chi, 1999; Mehrotra, 2008). When students perform such open-ended activities, it helps them: in constructive investigation, to overcome their preconceived notions, to accept and assess mistakes and to rethink or rework on ideas. Though preliminary, the analysis provides insights into planning design problem-solving activities for school students. Observations from this study can also be useful for teachers who wish to implement exploration-based making activities in schools. The work can be extended to understand if and how students reflect on the process of designing and arrive at generalist conclusions about factors that worked in their successful completion of the task.

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**Notes**

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https://www.instructables.com/id/Paper-Roller-Coasters-/

http://teachers.egfi-k12.org/wp-content/uploads/2018/06/Paper-Roller-Coasters-engineering-journal-and-velocity-calculations.pdf

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