HALF A CENTURY OF RESEARCH ON ALTERNATIVE CONCEPTIONS/MISCONCEPTIONS IN SCIENCE EDUCATION: WHAT HAS CHANGED?

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Studying students understandings of science phenomena is fascinating. A key element is unravelling ways of sense making as everyday experiences, language, overheard conversations are intertwined with what the teacher is saying. The teacher attempts to guide their learners to more scientific congruent ideas, the learner detours and goes through alleys slowly changing and every now and then producing what the teacher delights in hearing. The process is not quick enough for the teacher and convoluted for the learner. Is this a surprise? Once a learner sees ideas through the ‘lens of science’, some say it is hard to revert back, one has gone the threshold portal. But progressing through ‘the scientific lens’ is not trivial, after all, much is counterintuitive. What is reassuring for researchers is that there are identifiable, consistent and enduring ideas and pathways which form the cornerstones of alternative conceptions or misconceptions research. The solution is then to find ways through which these can be addressed. Overtime, experiments, technology, simulations and a range of tools have been identified and used. The successful interventions have been reported and some translated into systematic practice underpinning curricula. In this talk I will summarise the field, the contributions of my research team, from multimedia, Veritasium YouTube Channel to concept tools. A key finding which is often not reported is how students develop over their years of physics study, what are their trajectories of changing conceptions. If they don’t ‘overcome’ misconceptions in first year, can they ‘overcome’ them later on if not explicitly taught? We also offer a few different way of using and thinking about alternative conceptions, threshold concepts from the ‘troublesome knowledge’ tradition and LCT from linguistics.

Suggested Readings:

Sharma, M. D., Sefton, I. M., Cole, M., Whymark, A., Millar, R. M. and Smith, A. (2005) Effects of re-using a conceptual exam question in physics. *Research in Science Education*, **35**, 447-469.

Georgiou, H., and Sharma, M. D. (2015) Does using active learning in thermodynamics lectures improve students’ conceptual understanding and learning experiences? *European Journal of Physics,* **36(1)**, 015020.

Muller, D. A., Sharma, M. D. and Reimann, P. (2008) Raising cognitive load with linear multimedia to promote conceptual change. *Science Education* **92(2)**, 278-296.

Muller, D. A., Sharma, M. D., Eklund, J., Reimann, P. (2007) Conceptual change through vicarious learning in an authentic physics setting, *Instructional Science*, **35(6)**, 519- 533.

THE ATTRACTIVENESS OF A TOPIC APPROACH TO IMPROVING PEDAGOGICAL CONTENT KNOWLEDGE OF TEACHERS – LESSONS FROM RESEARCH IN SOUTH AFRICA

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It is thirty-four years since Lee Shulman drew the education research community's attention to the importance of explicit inclusion of content knowledge in both pre and in-service teacher education. To address this gap in our understanding of teacher knowledge he introduced the idea of pedagogical content knowledge (PCK) which was embraced with enthusiasm, particularly in the science education community. However, diverse interpretations about the nature of PCK and its relation to content knowledge followed. Since then, consensus has been sought to reach a unified understanding of the PCK construct through intensive discussions by researchers at two PCK summits held in 2012 and 2016, most recently resulting in a refined consensus model of PCK, published in 2019. PCK is theorised as powerful knowledge possessed by teachers, which enables them to transform content into a form that is easily understood by their students. It is tacit knowledge, which is thought to be acquired largely through experience. This talk provides a review of PCK models through the years culminating in a consideration of the power of conceptualising PCK at the topic level, known as Topic Specific PCK, or TSPCK. Consideration of PCK at the topic level has allowed researchers to look more closely at the some of the root causes of poor performance of South African students in science, which has been broadly blamed on teachers' poor content knowledge. The research described in this talk is driven by an attempt to improve both novice and experienced teachers' PCK through topic specific interventions. To measure the success of these interventions, validated pairs of instruments measuring CK and TSPCK have been designed to establish baseline knowledge of teachers in eight topics, two in physics and five in chemistry. The interventions have been very effective, improving teachers' TSPCK as well as their content knowledge. There is also evidence that the interventions enhance the teachers' ability to apply the tools used in the intervention to topics, which were not the subject of the interventions, now known as signature interventions. Further research has shown that pre-service teachers who have been exposed to signature interventions during their teacher education after qualification perform better than those who have not. The TSPCK research group at Wits University has produced 22 refereed articles and 35 masters and doctoral theses collecting evidence on topic specific PCK, the construct used as a lens to capture and measure PCK and investigate the effectiveness of interventions. The talk will also provide samples of data giving insights to the effectiveness of the approaches used in the research as well as the various methods of analysis used. The talk will conclude by looking at international collaborations currently under way and plans for future research.

Reference Papers:

Rollnick, M., Bennett, J., Rhemtula, M., Dharsey, N., & Ndlovu, T. (2008). The place of subject matter knowledge in pedagogical content knowledge: A case study of South African teachers teaching the amount of substance and chemical equilibrium. *International Journal of Science Education*, **30(10)**, 1365-1387. doi:10.1080/09500690802187025

Mavhunga, E., & Rollnick, M. (2013). Improving PCK of chemical equilibrium in pre-service teachers. *African Journal of Research in Mathematics, Science and Technology Education*, **17(1-2)**, 113-125.

Rollnick, M. (2016). Learning About Semi Conductors for Teaching—the Role Played by Content Knowledge in Pedagogical Content Knowledge (PCK) Development. *Research in Science Education*, **1-36**, doi:10.1007/s11165-016-9530-1

Mavhunga, E., & Rollnick, M. (2017). Implementing PCK Topic by Topic in Methodology Courses. *In Designing and Teaching the Secondary Science Methods Course* (pp. 149-170): Brill Sense.

PERSPECTIVES ON CONCEPTUAL CHANGE AND ITS NEXUS WITH IDENTITY

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After a brief overview of the perspectives on Conceptual Change and about the history of this multi-disciplinary research field, the talk will explore a novel, yet theoretically and methodologically thorny, research issue: the intersection between the conceptual change and identity.

Conceptual change is traditionally thought of as an individual cognitive phenomenon whilst identity development has usually been conceived of as a social construct involving the relationship between groups and individuals. This aspect raises important questions for how these constructs can be articulated in such a way as to be mutually illuminating.

The talk will illustrate and discuss significant approaches to forging productive connections between the research agendas of conceptual change and identity development. To pursue this goal the approaches will be reviewed and compared through a framework that allows for pointing out both the divergences and their common theoretical structure. This structure is argued to be potentially fruitful for orienting a program of further rigorous investigations of the nexus between conceptual change and identity.

Acknowledgment

The talk is based on the reflections and analyses that Tamer Amin, Mariana Levin and I carried out for edited volume entitled “Converging Perspectives on Conceptual Change: Mapping an Emerging Paradigm in the Learning Sciences” (Eds. T. Amin & O. Levrini, Routledge, 2018).

LOGIC IN SCHOOL MATHEMATICS: THE OUTSIDER AT THE WINDOW

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The infinite we shall do right away. The finite may take a little longer.
— Stanislaw Ulam

Ulam might as well have been talking of the school mathematics curriculum, which inexorably leads the student to Calculus at its end. Infinite sets (like the set of natural numbers and integers) and infinite objects (like real numbers, rays) are pervasive in mathematics from middle school onwards, though in an intuitive rather than axiomatic form. *Finite mathematics* makes short, almost apologetic, appearances. A syllabus unit titled *Mathematical reasoning* is often included. Typically it is about propositional logic, and students are trained in verifying if a given boolean formula is a tautology. Since this part is allotted only about 4% of the teaching time in the whole year (of Class XI in India) with 2% weightage in the final examination, it is not taken very seriously by all concerned, who have $\sin \alpha + \sin \beta$, conic sections, the binomial theorem etc. to worry about, and they are surely more difficult. Mathematical modelling is largely absent from school syllabi.

Now, with the realisation that discrete mathematics lies at the foundation of computation, a demand for it is heard, with logic included in the package. *Computational thinking* is the new paradigm, but though this is about enumeration, repetitive patterns and discrete modelling, it is not (yet) considered to be a part of the mathematics curriculum.

Yet, all through school, students learn deductive procedures in equational theories and employ deliberate means of reasoning in algebra and geometry. Interestingly, the little logic introduced tends to be propositional logic rather than the logic of quantification, while the latter is the form of logic unconsciously used by the student in mathematics. Leaving this implicit has serious drawbacks, as for example evidenced by students asked to solve the equation: $1/(x-1) = x/(x-1)$.

Logic remains the outsider in the mathematics classroom, not far away but gazing in from the window, watching these plays.

Logic is not only about deductive reasoning. Logic is also a conscious use of formal language, understanding truth relative to models, figuring out consequence, relating assertions to algorithms that check those assertions, and studying limits to reasoning.

In this talk, we observe that all these are already implicit, scattered here and there, in school mathematics, and suggest that there is reason to explicate these, for curricular and pedagogic purposes, as well as to enrich teacher knowledge. We discuss how granting first class citizenship to logic in school mathematics can help with computational thinking as well.

Suggested Readings:

1. John T Baldwin, *Model theory and the philosophy of mathematical practice*, Cambridge University Press, 2019.

2. Viviane Durand-Guerrier, Logic and mathematical reasoning from a didactical point of view, Thematic Group 4, European research in mathematics education III. available from http://www.mathematik.unitdortmund.de/~erme/CERME3/Groups/TG4/TG4_Guerrier_cerme3.pdf

3. Herbert Enderton, *A mathematical introduction to logic*, 2nd edition, Academic Press, 2001.

4. Susanna S Epp, Proof issues with existential quantification, In *Proof and Proving in Mathematics Education: ICMI Study 19 Conference Proceedings*, F. L. Lin et al eds., National Taiwan Normal University, 2009.

5. Susanna S Epp, Logic and discrete mathematics in the schools, In *Discrete Mathematics in the Schools*, *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, vol. 36.

6. Joseph G. Rosenstein, D. S. Franzblau, F. S. Roberts, Eds. *Providence*, RI: AMS Publications, 1997, 75-83.

SCHOOL SCIENCE, EQUALITY AND FAIRNESS

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Throughout the world science and technology, or STEM as it has come to be known, are seen as crucial instruments in ensuring national prosperity. At the same time there is a consciousness that the products of science and technology should be directed towards the public good, hence the policy coda of Responsible Research & Innovation (RRI) which underpins funding of S&T research and education in the European Union. This linking of social issues to science - socio-scientific issues in educational terms - has always faced epistemological problems. These problems include the focus on the Mertonian norms of science as objective and disinterested, the prevalence of empiricist and positivistic methods in science practise, and the ideological sway of Hume's naturalistic fallacy or the 'is-ought' dichotomy. Indeed some educationalists have argued effectively that science as a discipline has a distinctive space in the school curriculum with a unique set of concepts and principles (Hirst & Peters, 2011)

I shall argue that interpretations of Enlightenment rationality have hampered the development of socio-scientific issues and the gearing of science education to social justice. Rather than argue for a bolt-on connection between science and society, underpinning so-called Vision I and Vision II approaches (Roberts & Bybee, 2014) I claim that the practise of science can only flourish through an understanding of social justice at its core. Prevalent neoliberal formulations of science and society mean that S&T research and development skims over deep and structural injustices.

There are two theoretical positions I shall draw on, with examples, to argue that science practise and learning cannot be decoupled from questions of social justice. Critical Realist metatheory (Collier, 1994; Levinson, 2018a) has the reality of human emancipation at its core. Taking the world as ontologically real (the intransitive dimension) and epistemologically relativist (the transitive dimension) – what is is not the same as what is known - Critical Realism considers natural phenomena as open systems to be investigated. It finds an approach to science practise between naïve realism and empiricism, buttressed by an appeal to judgemental rationality. Theories about the nature of reality can be judged according to valid criteria of truth. At the same time stratification and emergence can generate explanations through causal mechanisms in diverse disciplines from the physico-chemical to the socio-economic. The explanation of events is thus inter-disciplinary.

Secondly Levinas's ethics of the refusal of subjectivity allows us to recognise difference and diversity, that Nature can be studied from a different perspective from the dominant subjective 'I', a hangover from

Enlightenment rationality. From this perspective I create a picture of the non-presumptive and knowledgeable science teacher (Levinson, 2018b). If social justice is intrinsic to science education then it must also be at the heart of pedagogy. Finally I draw on the 'story' demonstrating how the personal and political are interwoven in understanding scientific ideas through interlocking narratives (Levinson, 2009). My conclusion is that science teaching should focus on explaining events in an interdisciplinary manner which not only couples science to the social but also deepens understanding of core scientific concepts.

References:

Collier, A. (1994). Critical Realism: *an introduction to Roy Bhaskar's philosophy*. London:Verso.

Hirst, P.H. & Peters, R.S. (2011). *The logic of education*. London and New York: Routledge.

Levinson, R. (2009) The manufacture of aluminium and the rubbish-pickers of Rio: building interlocking narratives, *School Science Review* 90 (333): 119-124

Levinson, R. (2018a). *Real*ising the school science curriculum. *The Curriculum Journal*, 29(4): 522-537.

Levinson, R. (2018b). I know what I want to teach but how can I know what they are going to learn? In Bryan, L. & Tobin, K. (Eds) *Critical Issues and Bold Visions for Science Education: The Road Ahead.* Boston: Brill., pp 59-74.

Roberts, D.A. & Bybee, R.W. (2014). Scientific literacy, science literacy and science education. In N.G. Lederman & S.K.Abell (Eds.) *Handbook of Research on Science Education*. New York: Taylor & Francis. Chapter 27.

INDUCTING CHILDREN IN THE EPISTEMOLOGY OF MODELING

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A central aim of education is to help learners understand how knowledge is articulated in the disciplines. Contemporary perspectives in the learning sciences emphasize that knowing emerges from interactions among discipline-specific practices that generate an ensemble of concepts, along with ways of thinking about their significance in light of imagined and experienced critique. To create conditions that support this kind of learning, I work with teachers to design learning ecologies in which children in the elementary grades are initiated into approximations of the practices employed by STEM professionals to germinate, revise, and maintain knowledge. The design of learning ecologies includes making informed bets about STEM practices that can be robustly and fruitfully approximated in classrooms. These commitments are accompanied by conjectures about (a) how these practices interact to develop new knowledge, (b) the kinds of tasks and means of articulation that will support this hypothetical development, and (c) how to establish and maintain settings in which children can participate in both the production and critique of these emerging concepts and practices. All of these aspects of design are orchestrated by teachers, so teaching and learning are viewed as coupled, a perspective in the learning sciences that is most directly advanced by an approach known as design research. I illustrate this epistemic perspective on learning with two examples of design research conducted to introduce children to the signature practice of the sciences, modeling. The first example traces fifth- and sixth-grade students' (ages 10,11) induction into statistical practices of visualizing, measuring, and modeling variability. Engaging with these practices supported students' development of new ways of conceiving of samples as simultaneously a distribution of outcomes from a portion of a repeated stochastic process (a sample) and as distributed (a sampling distribution). These experiences initiated a new way of thinking about inference under conditions of uncertainty, an essential form of inference in sciences. The second example describes how young (ages 6,7) and older students (age 11) experienced the essential dialectic between performative and representational aspects of modeling as they noticed and explained similarities and differences among local ecosystems (prairie, forest, and pond). On the performative side, children worked to achieve a material grip on ecosystems by designing investigations, choosing appropriate tools, and developing measures to make the workings of these systems visible. On the representational side, children invented and revised inscriptions of material arrangements and established circulation (mutual reference) among these inscriptions to develop understanding of ecosystem functioning. I conclude with suggestions for productive new directions in research to support children's participation in the epistemology of modeling.

Related References:

Lehrer, R. (2017). Modeling signal-noise processes supports student construction of a hierarchical image of sample. *Statistics Education Research Journal*, *16* (2), 64-85.

Lehrer, R., & English, L. (2018). Introducing children to modeling variability. In Ben-Zvi, D., Makar, K., & Garfield, J., (Editors). *International Handbook of Research in Statistics Education* (pp.229-260). Voorburg, the Netherlands: Springer.

Lehrer, R., Kim, M. J., & Jones, S. (2011). Developing conceptions of statistics by designing measures of distribution. *International Journal on Mathematics Education* (ZDM), *43* (5), 723-736.

Lehrer, R., & Schauble, L. (2012). Seeding evolutionary thinking by engaging children in modeling its foundations. *Science Education*, *96*(4), 701-724.

Lehrer, R., & Schauble, L. (2017). Children's conceptions of sample in local ecosystem investigations. *Science Education*, 701-724. DOI**:** 10.1002/sce.21297

Lehrer, R., & Schauble, L. (2018). The dynamic material and representational practices of modeling. In T. Amin and O. Levrini (Eds.), *Converging perspectives on conceptual change* (pp. 163-170). New York: Routledge.

Lehrer, R., Kim, M., & Schauble, L. (2007). Supporting the development of conceptions of statistics by engaging students in modeling and measuring variability. *International Journal of Computers for Mathematics Learning, 12,* 195-216.

Lehrer, R., Schauble, L., & Lucas, D. (2008). Supporting development of the epistemology of inquiry.*Cognitive Development*, *24*, 512-529.

BUILDING HYBRID MINDS: PEDAGOGY IN THE AGE OF LEARNING MACHINES

*Machines that learn and discover are now a critical part of science practice.
How can science education adapt to this change?*

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There is now consensus in the philosophy of science that building *explicit* models of natural phenomena is a core practice that supports scientific discovery. Following this, pedagogy based on the building of explicit models is central to the design of science learning.

However, discovery practices in contemporary science have changed — towards building *opaque* computational models, which do not provide explicit explanations. Such models dominate in the fast-developing *engineering sciences* (bioengineering, material science, systems/synthetic biology, robotics, artificial intelligence etc.), where the key objective is not developing explicit accounts, but *building*, *controlling* and *manipulating* novel synthetic artifacts, which mimic complex natural phenomena (neurons, metabolic networks, organism behavior, shape-memory etc.).

In most engineering science situations, where such synthetic artifacts are built in tandem with computational models, it is not possible to develop explicit accounts, even in principle. This is because such computational models are built as a last cognitive resort — when the non-linear interactions that are part of complex target systems are beyond both standard explicit modeling approaches and the human imagination. Opaque computational approaches (such as machine learning) are *required* to manage the overwhelming cognitive complexity in such situations.

This discovery practice has created a strange knowledge crisis, where machines built by humans discover patterns that humans cannot perceive. Machines also generate and ‘meld’ such patterns, to design novel solutions humans cannot imagine. Further, there is now a fast-developing ‘hybrid intelligence’ effort, based on interactive ‘human-in-the-loop’ video games, which allow two kinds of human-machine hybrids. One, such games allow crowds of novice players on the web to build novel scientific models of complex phenomena (such as protein folds, quantum computing, RNA, and neuronal networks), using their *tacit* sensory-motor capabilities, which are not available for conscious tracking and articulation. Second, in parallel, machine learning systems extract patterns from players’ tacit moves, and generate novel models. Such hybrid systems create cognitive black boxes, where humans don’t know what they are teaching the machine, and the machines don’t know what they have learned. But they can together generate useful predictions and designs.

I will argue that the emergence of such hybrid modeling systems for discovery, and more broadly, *machines that learn and discover*, is a radical cognitive shift — similar to the emergence of tool use, language and literacy. These older cognitive shifts emerged across thousands of years, allowing learning and education systems to evolve in parallel. The ongoing shift to learning and discovery machines is occuring in internet time. To prepare students for this radical shift, science education needs to develop pedagogies that can evolve and adapt quickly, in step with fast developments in this domain.

Traditional pedagogical approaches (such as showing and telling) are not enough to adapt to this radical transition. The closest pedagogical process that appears suitable as a starting point is the *building* of prototypes, which is now promoted extensively through maker-spaces and tinkering labs. However, these building initiatives are not designed to support the building and manipulation of machine models for discovery. They are intended to kick start innovation cultures, where the building emphasis is on making of useful artifacts.

Learning to build and manipulate machines for discovery requires a new pedagogy of building. This is a very challenging and murky design problem. I will outline two directions the LSR group is pursuing to address this problem.

References:

Chandrasekharan, S., Nersessian, N.J. (2015). Building Cognition: the Construction of Computational Representations for Scientific Discovery. *Cognitive Science*, 39, 1727–1763.

Chandrasekharan, S., Nersessian, N.J. (2018). Rethinking correspondence: how the process of constructing models leads to discoveries and transfer in the bioengineering sciences. *Synthese*, DOI 10.1007/s11229-017-1463-3

Chandrasekharan, S. (2009). Building to discover: a common coding model. *Cognitive Science*, 33 (6), 1059-1086.

*Aurigemma, J., Chandrasekharan, S., Newstetter, W., Nersessian, N.J. (2013). Turning experiments into objects: the cognitive processes involved in the design of a lab-on-a-chip device. *Journal of Engineering Education*, 102(1), 117-140.
*All authors contributed equally

Chandrasekharan, S. (2016). Beyond Telling: Where New Computational Media is Taking Model-Based Reasoning. In *Model-Based Reasoning in Science and Technology, Volume 27 of the series Studies in Applied Philosophy, Epistemology and Rational Ethics*, pp 471-487, Springer, Heidelberg.

Chandrasekharan, S. (2014). Becoming Knowledge: Cognitive and Neural Mechanisms that Support Scientific Intuition. In Osbeck, L., Held, B. (Eds.). *Rational Intuition: Philosophical Roots, Scientific Investigations*. Cambridge University Press. New York.

Rahaman, J., Agrawal, H., Srivastava, N., Chandrasekharan, S. (2018). Recombinant enaction: manipulatives generate new procedures in the imagination, by extending and recombining action spaces. *Cognitive Science*, 42(2), 370–415.

CONCEPTS, METAPHORS AND CONCEPTUAL CHANGE IN SCIENCE LEARNING: A CONCEPTUAL METAPHOR PERSPECTIVE

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One of the main goals of the field of cognitive linguistics is to identify the many subtle conceptual construals implicit in linguistic choices. This has included identifying vast patterns of metaphorical construals in everyday language use not usually recognized as metaphorical. For example, time is conceptualized in terms of movement in space (as in *winter is coming; I can't wait until we get to summer*) and sometimes as a resource (as in *don't waste more time*; *time is running out*). Other examples (among many identified in the literature) are emotional states construed as containers (as in *I'm in a bad mood*; *he's in love*); goals are construed as destinations (as in *I'm moving in the right direction in my career*); and causes are construed as forces (as in *the performance lifted the crowd to their feet*). The theory of conceptual metaphor, developed based on these analyses, makes two central claims: that metaphorical expressions reflect underlying systematic mappings between conceptual domains; and that abstract conceptual domains are understood metaphorically in terms of more concrete, experiential knowledge. This more concrete knowledge is in the form of image schemas – that is, abstractions from repeated sensorimotor experiences - such as containment, moving objects, path, and forced movement. These claims offer those of us interested in science learning a way of thinking about how an understanding of abstract scientific concepts might be acquired. We have shown that even scientific concepts as abstract as the concept of energy are construed metaphorically in terms of image schemas: energy exchange can be construed as movement of a substance (as in *put energy into the gas*); forms of energy can be construed as containers (as in *the energy was stored in potential chemical energy*); and energy conservation book-keeping can be done construing energy in terms of a part-whole schema (as in *part of the system's energy was in the kinetic energy of the particles*). A lot of research on science learning and instruction over the last decade or so has used ideas from the theory of conceptual metaphor. This work has shown the relevance of a conceptual metaphor perspective to characterizing expert scientific understanding and reasoning, assessing and characterizing learner conceptions, describing the process of conceptual change, selecting and designing instructional representations and analogies and designing science curricula. In this talk, I will review this research, highlighting in particular how a conceptual metaphor perspective contributes to understanding conceptual change in science learning and what new questions it suggests. But I will argue that for substantial progress to be made in using a conceptual metaphor perspective to understand conceptual change, we need a clearer account of where conceptual metaphors fit in a theory of concepts. Specifically, I will argue that it is useful to integrate the perspective of conceptual metaphor with a view of concepts that emphasizes both how a concept refer to things in the world and participates in an inferential network.

References:

Amin, T. G. (2018). Representation, concepts and concept learning. In T. G. Amin and O. Levrini (Eds.), *Converging perspectives on conceptual change: Mapping an emerging paradigm in the learning sciences*. London, UK: Routledge.

Amin, T. G., Jeppsson, F, and Haglund, J. (Eds.) (2015). Special issue entitled "Conceptual metaphor and embodied cognition in science learning". *International Journal of Science Education*, 37(5-6), 745-991.

Amin, T. G. (2015). Conceptual metaphor and the study of conceptual change: Research synthesis and future directions. *International Journal of Science Education*, *37*(5-6), 966-991.

Amin, T. G. (2009). Conceptual metaphor meets conceptual change. *Human Development*, *52*(3), 165-197.

LEVELS OF ABSTRACTION IN SCHOOL ARITHMETIC

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In my article of A critique of the structure of U.S. elementary school mathematics (Ma, 2013), I discussed two organization types of elementary school mathematics. One has a "core-subject structure," and the other "a strands structure." I used the Chinese elementary school math standards before 2001 and the U. S. NCTM Standards as the examples to illustrate a comparison between them (Figure 1):

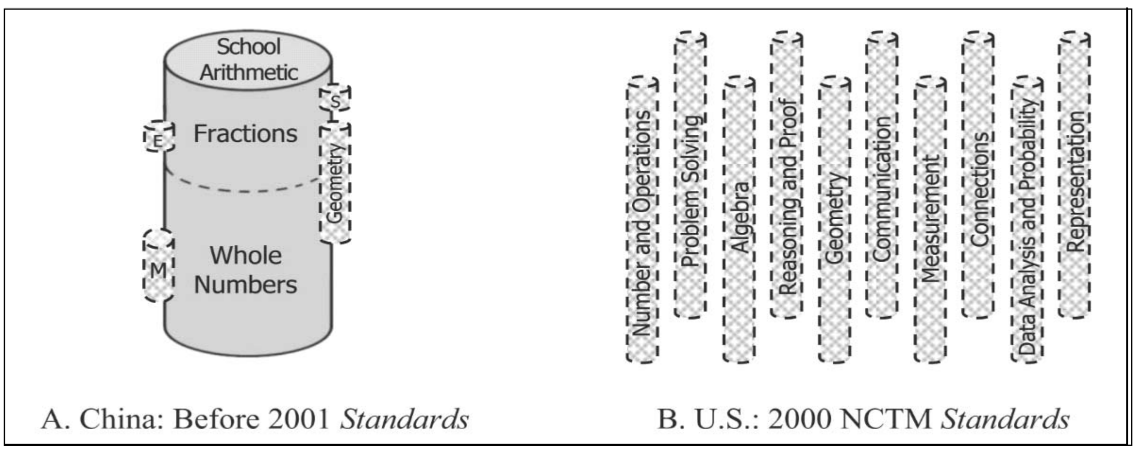

Figure 1. Two organizations of elementary school mathematics.

Example A has a "core-subject structure." The large gray cylinder in the center represents school arithmetic. Its solid outline indicates that it is a "self-contained subject." School arithmetic consists of two parts: whole numbers and fractions. Knowledge of whole numbers is the foundation upon which knowledge of fractions is built. The smaller cylinders represent the four other components of elementary mathematics, shown according to the order in which they appear in instruction. These are: measurement (M), elementary geometry, simple equations (E), and simple statistics (S). The core subject of elementary mathematics is what I call "school arithmetic." The subject of school arithmetic was constructed following the model of Euclid's *The Elements*. Although it took several decades to be comprehensively developed, its feature of being self-contained never changed. That feature ensures the consistency of elementary mathematics contents with school arithmetic as the core.

Example B has a "strands structure." Its components are juxtaposed, but not connected. Each of the ten cylinders represents one standard in *Principles and Standards for School Mathematics.* No self-contained subject is shown. This type of structure has existed in the U.S. for almost fifty years, since the beginning of the 1960s. The number and the components in a "Strands Structure" can be frequently changed and replaced, according to the different visions of the education policy makers. In this way, the consistency becomes a "luxury" hard to attain.

In the article The Theory of School Arithmetic: Whole Numbers (Ma & Kessel, 2018), Kessel and I pointed out that the idea of "unit one" is the fundamental concept on which the subject of school arithmetic is built on. We also addressed on several stages that the concept of "unit one" evolves in whole numbers, and how they may inspire students' abstractive thinking step by step.

In this speech I would like to expand the issue into fractions. A more detailed description of the evolution of the concept of "unit one" in school arithmetic, from concrete to abstract, from simple to sophisticated, will be discussed.

1) One-digit numbers. Addition and subtraction exclusively with one-digit numbers;
2) Multi-digit numbers. Addition and subtraction with whole numbers;
3) Multiplication and division with whole numbers;
4) The four fundamental operations of whole numbers;
5) Fractions. Addition and subtraction with fractions;
6) Multiplication with fractions;
7) Division with fractions;
8) The four fundamental operations of fractions.

The Fig. 2 presents the eight levels of abstraction levels of the concept of unit one in school arithmetic:

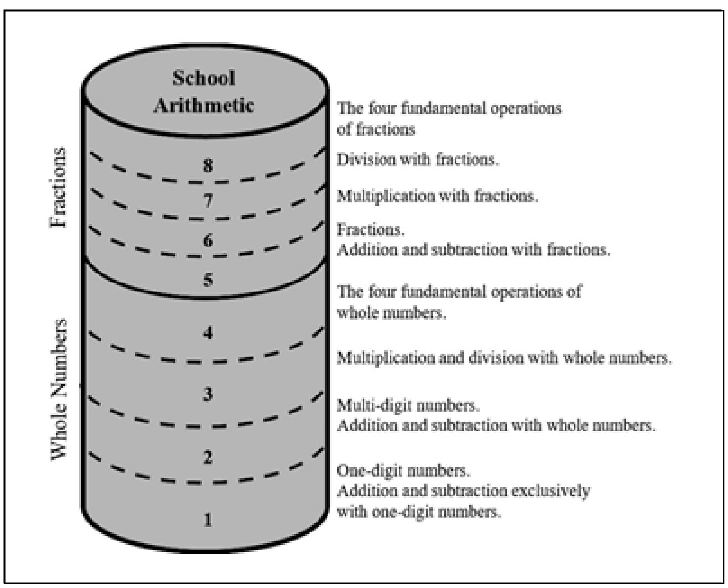

Figure 2. Eight abstraction levels of the concept of unit one in school arithmetic

I will use word problems to illustrate these eight levels of abstraction.

References

Ma, L. (2013). A critique of the structure of U.S. elementary school mathematics. *Notices of the American Mathematical Society*, *60*(10), 1282–1296, http://www.ams.org/notices/201310/fea-ma.pdf.

Ma, L. & Kessel (2018). The Theory of School Arithmetic: Whole Numbers. *Building the Foundation: Whole Numbers in the Primary Grades.* (Bussi, M. & Sun, X. Editors), Springer, Switzerland.

National Council of Teachers of Mathematics (NCTM) (2001). Principles and Standards for School Mathematics